Discounted Cash Flow: Application and Misapplication

Hal Heaton, PhD
The Income Approach to Valuation

In the income approach (specifically the discounted cash flow approach) value is determined by discounting cash flows:

\[
Value = \frac{CashFlow_1}{(1+k)^1} + \frac{CashFlow_2}{(1+k)^2} + \ldots + \frac{CashFlow_n}{(1+k)^n}
\]

The discount rate or “k” in the equation is the cost of capital.
Perpetual Growth Formula

• If the cash flow stream grows forever at a constant rate “g” then the formula simplifies to

\[
Value = \frac{\text{CashFlow}_1}{(1+k)^1} + \frac{\text{CashFlow}_2}{(1+k)^2} + \ldots + \frac{\text{CashFlow}_n}{(1+k)^n}
\]

\[
= \frac{\text{CashFlow}_1}{k-g}
\]
Yield Capitalization

- If the company is just expected to earn its cost of capital on average in future years, then the formula can be expressed as

\[
Value = \frac{NOI}{k} = \frac{CashFlow_1}{(1+k)^1} + \frac{CashFlow_2}{(1+k)^2} + \ldots + \frac{CashFlow_n}{(1+k)^n}
\]

- “NOI” is net operating income

- “Cash flow” in this case is the payout ratio times NOI, i.e. the portion of NOI that is paid out every year to the investors.

- Note that, by assumption, in this case: \( Value \times k = NOI \).
Be Careful of an Erroneous Formula

- **Erroneous Value** = Net Operating Income (NOI) / [Cost of Capital - Inflation (growth)]
  - NOI/(k-g)
- **Proof of Error**
  - V = CF/(k-g)
  - CF = NOI (PO)
  - g = (1-PO)ROI
  - ROI=k
  - V = NOI(PO)/[k-(1-PO)k] = NOI/k **not** NOI/(k-g)
- **Bank deposit example**
  - Deposit $1000 earning 10% and withdraw 30% of interest every year
  - NOI= $100
  - Cash flow = $30
  - G = 7%
  - Value = NOI/k = $100/10% = $1000 or CF/(k-g) = $30/(.10-.07) = $1000
  - **NOT** $100/(.10-.07) = $3,333
Be Careful! The Discount Rate Must Match the Cash Flow Being Discounted

The standard textbook cash flow (After-tax):

Revenues
- Cash Operating Expenses
- Depreciation
= Earnings Before Interest and Taxes (EBIT)
- Taxes on EBIT
= Net Operating Profits After Tax (NOPAT)
+ Depreciation
- Capital Expenditures
- Increases in Working Capital
= Free Cash Flow

*Weighted Average Cost of Capital (WACC) = \([w_d \times k_d \times (1-T)] + [w_e \times k_e]\)*

Adjustment for tax-deductibility of interest is in discount rate.
Be Careful! The Rate Must Match the Cash Flow Being Discounted (continued)

The regulatory model:

Revenues
- Cash Operating Expenses
- Depreciation
= Earnings Before Interest and Taxes (EBIT)
- Taxes on EBIT
= Net Operating Profits After Tax (NOPAT)
+ Depreciation
+ Tax savings from interest deductions
- Capital Expenditures
- Increases in Working Capital
= Free Cash Flow

*Weighted Average Cost of Capital (WACC) = \([w_d \times k_d] + [w_e \times k_e]\)*

Adjustment for tax-deductibility of interest is made in the cash flows.
Effective tax rate mixing GAAP and IRS

• Some publications cite “effective tax rate”
  • Differs from publication to publication
  • Vague definition but usually calculated as $\frac{\text{Taxes}}{\text{GAAP Pretax Income}}$

• **Does not reflect tax impact of $1 more of income**
  • Can be seriously distorted by company situation
    • Investment tax credits, net operating loss carryforwards, losses from other businesses
    • Accelerated depreciation, other differences between IRS and GAAP income
  • Does not reflect cash taxes that will be paid going forward
    • A dollar of interest will save the marginal tax rate (e.g. IRS tax rate) not the effective tax rate of taxes
    • A dollar of income will be taxed at the IRS tax rate not the “effective tax rate”

• Use **IRS Marginal Tax Rate**
Using pretax cash flows

- Problem most severe for short term cash flows

<table>
<thead>
<tr>
<th>Single Cash Flow</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings Before Interest, Depn &amp; Amortzn (EBITDA)</td>
<td>$240</td>
<td>$240</td>
<td>$240</td>
</tr>
<tr>
<td>Depreciation &amp; Amortization (DA)</td>
<td>$50</td>
<td>$10</td>
<td>$150</td>
</tr>
<tr>
<td>Profit Before Tax</td>
<td>$190</td>
<td>$230</td>
<td>$90</td>
</tr>
<tr>
<td>Tax at 40%</td>
<td>$76</td>
<td>$92</td>
<td>$36</td>
</tr>
<tr>
<td>Optg Profits After Tax</td>
<td>$114</td>
<td>$138</td>
<td>$54</td>
</tr>
<tr>
<td>Add Back DA</td>
<td>$50</td>
<td>$10</td>
<td>$150</td>
</tr>
<tr>
<td>Subtract Capital Expenditures</td>
<td>$64</td>
<td>$48</td>
<td>$104</td>
</tr>
<tr>
<td>After Tax Cash Flows</td>
<td>$100</td>
<td>$100</td>
<td>$100</td>
</tr>
<tr>
<td>Value at 10% After-Tax</td>
<td>$91</td>
<td>$91</td>
<td>$91</td>
</tr>
<tr>
<td>EBITDA</td>
<td>$240</td>
<td>$240</td>
<td>$240</td>
</tr>
<tr>
<td>Capital Expenditures</td>
<td>$64</td>
<td>$48</td>
<td>$104</td>
</tr>
<tr>
<td>Pre-Tax Cash Flows</td>
<td>$176</td>
<td>$192</td>
<td>$136</td>
</tr>
<tr>
<td>Required Before-Tax Discount Rate to Achieve Correct Value</td>
<td>93.6%</td>
<td>111.2%</td>
<td>49.6%</td>
</tr>
</tbody>
</table>
Using pretax cash flows

- 10% after-tax = \( \frac{10}{1-40\%} \) = 16.7% pre-tax

Annuity formula only works if Depn = Cap Ex

### Perpetual Annuity

<table>
<thead>
<tr>
<th>Earnings Before Interest, Depn &amp; Amortzn (EBITDA)</th>
<th>Scenario 1: Flat Annuity</th>
<th>Scenario 2: CF Growing at 4%</th>
<th>Scenario 3: Flat Annuity, CapEx=Depn</th>
</tr>
</thead>
<tbody>
<tr>
<td>$240</td>
<td>$240</td>
<td>$240</td>
<td></td>
</tr>
</tbody>
</table>

| Depreciation & Amortization (DA)                  | $50                      | $50                        | $73                             |
|                                                  |                          |                            |

| Profit Before Tax                                 | $190                     | $190                       | $167                            |
| Tax at 40%                                        | $76                      | $76                        | $67                             |

| Optg Profits After Tax                            | $114                     | $114                       | $100                            |
| Add Back DA                                       | $50                      | $50                        | $73                             |
| Subtract Capital Expenditures                     | $64                      | $64                        | $73                             |

| Initial After Tax Cash Flow                       | $100                     | $100                       | $100                            |

| Value at 10% After-Tax                            | $1,000                   | $1,667                     | $1,000                          |

| EBITDA                                           | $240                     | $240                       | $240                            |
| Capital Expenditures                              | $64                      | $64                        | $73                             |

| Initial Pre-Tax Cash Flow                         | $176                     | $176                       | $167                            |

| Required Before-Tax Discount Rate to Achieve Correct Value | 17.6% | 14.6% | 16.7% |
Matching cost approach: physical, functional, economic obsolescence

- Physical Obsolescence
  - Must be reflected in sufficient capital expenditures to maintain revenue forecast

- Functional Obsolescence
  - New equipment lower cost
  - And more capacity
  - And smaller in physical size

- Economic Obsolescence
  - Best evidence of economic obsolescence is that the return is insufficient to justify cost
Matching cost approach: physical obsolescence

- Capital expenditures must be sufficient to support growth in revenues
  - If, for example, capital expenditures grow at 2% but revenues (and hence cash flows) grow at 4%, return on investment grows without limit!
  - Particularly troublesome in terminal value calculation
    - Growth in Gordon Growth Model \( V_T = \frac{CF_{T+1}}{(k-g)} \) must be supported by necessary capital expenditures, otherwise expected return increases forever

- Example:
  - Appraiser forecasted earnings and net cash flow in Gordon Growth Model to grow at 2.6% per year
    - Depreciation add back in net cash flow calculation = $2.2 billion
    - Capital expenditures in net cash flow calculation = $1.7 billion
    - Net book value falls forever but earnings rise forever => return on book value rises forever
    - IRS does not allow depreciating assets below zero
Matching cost approach: functional obsolescence

<table>
<thead>
<tr>
<th>Last Year Revenue for Average Customer</th>
<th>Amount</th>
<th>Income Statement Accounting numbers</th>
<th>Average Customer Revenue in 5 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>$600</td>
<td>Revenues</td>
<td>$800</td>
</tr>
<tr>
<td>Expenses to provide service</td>
<td>200</td>
<td>Maintenance, operations, marketing, etc. expenses</td>
<td>220</td>
</tr>
<tr>
<td>Phone</td>
<td>150</td>
<td>Manufacturer expense</td>
<td>160</td>
</tr>
<tr>
<td>Government</td>
<td>50</td>
<td>Universal Service Fund, income taxes, property taxes, etc.</td>
<td>60</td>
</tr>
<tr>
<td>Spectrum</td>
<td>100</td>
<td>Interest, profit (i.e. cost of capital for usage of spectrum)</td>
<td>130</td>
</tr>
<tr>
<td>Network (PPE) Usage of 4G network = 10 gig/year</td>
<td>100</td>
<td>Interest, depreciation, profit (i.e. cost of capital for usage of 4G network)</td>
<td>13</td>
</tr>
<tr>
<td>Content</td>
<td>0</td>
<td>Expense of content providers</td>
<td>100</td>
</tr>
<tr>
<td>New network usage (reflecting capital expenditures for new, lower cost, higher capacity 5G technology installed during 5 years) = 90 gig/yr</td>
<td>0</td>
<td>Interest, depreciation, profit (i.e. cost of capital for usage of 5G network)</td>
<td>117</td>
</tr>
</tbody>
</table>
Growth in Cash Flows v. Growth in Dividend Growth Model for Cost of Equity

• Cash flows should reflect only the cash flows generated by the existing assets on the lien date
  • Most growth in future revenues stems from additional capital expenditures beyond that necessary to replace the productive capacity of the existing assets
  • For property tax purposes, an appraiser is not valuing the business
    • The business reflects future value from assets which don’t yet exist

• Total future growth in earnings (analyst estimates) reflect total future growth—i.e. growth from existing assets and growth from future assets

• Cash flows from existing assets will generally grow only at the rate of inflation
Dividend Growth Model

• *Estimated Required Return on Equity = Dividend Yield + Expected Growth*
  • \((\text{Dividend} / \text{Price}) + g\)

• Represents an estimate of return expected by investor for a particular stock

• Current price better captures current economic condition than 5 year historical beta calculation

• Be careful as to what data is being used
Dividend Growth Model

• *Do not use an average of dividend and earnings growth*

• Dividend growth much slower
  • Companies are shifting to share repurchases and away from dividends
    • Tax reasons
    • Greater flexibility to respond to adverse economic conditions

• Earnings per share growth rate better reflects cash flow growth of company
  • Captures full cash distribution capability, not just dividend choice
  • Captures effect of share repurchases through impact on earnings per share

• *Use earnings per share growth rate in dividend growth model*
Note: Number of shares falling => share repurchases
2011 to 2015 dividend growth rate = 2.2%, earnings per share growth rate = 5.3%
Growth of cash flows of existing assets

• Usually only inflationary growth
• Must still subtract increased capital expenditures since the cost of capital expenditures will increase with inflation
• Must still subtract increased working capital needs since current assets and liabilities will tend to grow with revenues (inflation)
Depreciation

• In the income approach, the appraiser values only cash flows from the existing assets
  • Depreciation is not a cash flow
  • But it does affect taxes which are a cash flow

• Companies keep two sets of books
  • Generally Accepted Accounting Principles (Annual Report, SEC 10K)
    • E.g. straight line depreciation
  • IRS accounting
    • Modified Accelerated Cost Recovery System (MACRS)

• GAAP statements do not reflect cash flows
  • Taxes reflect both cash taxes and deferred taxes

• It is cash flows (cash taxes) that should be determined in cash flow
  • IRS depreciation and cash taxes
  • Alternatively, GAAP numbers need to reflect adjustments to determine actual cash flows
    • If deferred tax added back, recognize that increased deferred tax will reduce rate base of rate-regulated utilities and revenue cash flow must drop accordingly
Direct Capitalization

• Rate is calculated by finding other comparable properties which have recently sold and dividing ‘income’ by the sales price for each comparable.

• ‘Income’ can be almost anything (gross rents, net rents, EBIT, NOI, etc.) as long as it is calculated the same way for all comparable properties and the subject property.

• Must be very careful to choose truly comparable properties (same age, condition, location, financing, etc.)

• Value is determined by taking the ‘rate’ from the comparable properties and dividing it into the ‘income’ of the subject property

• This ‘rate’ is not a cost of capital; in fact this approach is mathematically a market approach or sales comparison approach rather than an income approach.
Stock Market “Direct Capitalization”

• Uses stock market EP (1/PE) ratios for cost of equity
• Not discussed in any appraisal text
• Mathematically a stock and debt approach
• Because it is a stock and debt approach, it will include all intangible values
• Includes liquidity value
  • Stocks and bonds are liquid, property is not
‘Comparables’ Are Securities NOT Property

• Most risk measurements (P/E ratios, betas) are computed from traded stocks of companies selected as comparable
• Betas are calculated by comparing returns on stocks versus the S&P 500 stocks
• Return estimates from the CAPM and Dividend Growth Model are required returns on traded securities
• Operating property is not comparable to traded securities
  • Much less liquid ....
Illiquidity

- Liquidity refers to the ability to sell an investment easily, quickly, and at low cost
  - A *liquidity discount* refers to the lower value of an illiquid asset compared to a liquid asset of similar risk
  - A *liquidity premium* refers to the higher return that investors will require for an illiquid asset
- Liquidity became critical during last recession and remains a critical issue
  - Ability to generate cash to meet obligations critical
  - Treasury bills are offering virtually zero interest
  - 30-day Treasury bills briefly offered *negative* interest!
Illustration

• Build a power plant ...
  ◦ Cost $800 million

• Hire managers, train a work force, market to obtain contracts and customer base
  ◦ Cost $200 million

• If property generates $100 million per year and 10% is required rate
  ◦ Value = $1000 = $100 / 10%

• Issue and sell stock (debt) claims on the property
  ◦ Incur substantial costs to issue
  ◦ Incur ongoing costs to stay listed

• Compare owning the property versus buying shares ....
Illustration (Continued)

• If you own the property you must
  ◦ Have substantial knowledge of how to operate facility, market products or services
  ◦ Worry about hiring, firing, training
  ◦ Take care of all regulatory, licensing, disclosure, and other issues

• If you own the property you do not have limited liability
  ◦ Environmental, accident, other litigation may lead to losing other assets

• Selling property takes time, expense, ...
Illustration (Continued)

- If you buy the shares
  - You do not have to know anything about managing, operating, marketing, regulations ....
  - You can buy a few shares or a lot
    - Easy to diversify
- Shareholders have absolute limited liability
- You can turn your ownership into cash in seconds with the click of an icon
- You own future assets = growth
- You can capture value from
  - Intangibles
  - Net present value from assets that do no even exist on the lien date

- Which you would rather own?
If the shares sell for, say, $1.5 billion due to all the conveniences and advantages
- The property is still only generating $100 million a year
- Hence the discount rates extracted from stock and bond data must be lower than 10%
- Lower rates show up in any estimated return for securities
  - Cost of debt
  - Capital Asset Pricing Model Estimate
  - Dividend Growth Model Estimate
Assessors recognize need for liquidity adjustments

  - “Most financial assets are liquid. Real estate and most business assets, however, are relatively illiquid, and real estate investors must be compensated for this reduced liquidity.”

  - “The argument based on lack of liquidity is a much stronger one. There is no question that financial assets are significantly more liquid than real estate assets. ... An adjustment for lack of liquidity can be made in two ways: (1) consider lack of liquidity as an added risk factor and add a premium for it to the cost of equity estimated by the CAPM; or (2) value the real estate asset using the CAPM/WACC without any liquidity adjustment, and then apply a liquidity discount to the estimated value.”
Extensive finance research verifies liquidity impact on value...

- “Liquidity (or marketability) is a key attribute of capital assets, and it strongly affects their pricing...investors prefer to commit capital to liquid investments, which can be traded quickly and at low cost whenever the need arises. Investments with less liquidity must offer higher expected returns to attract investors.” [“Liquidity, Asset Prices and Financial Policy,” Y. Amihud and H. Mendelson, Financial Analysts Journal (Nov./Dec. 1991), p. 56]

- “...liquidity-increasing financial policies may increase the value of the firm. This was demonstrated for our numerical example. ... If the spread is reduced to 0.486% [from 3.2%] (as in our low-spread portfolio group), our estimates imply that the value of the asset would increase to $75.8, about a 50% increase....” [“Asset Pricing and the Bid-Ask Spread,” Y. Amihud and H. Mendelson, Journal of Financial Economics, 17(2), p. 246.]

- “Our study contributes to the academic literature since we believe we offer the cleanest and most precise measures of the value of liquidity. Due to the unique experimental design inherent in REITs, especially the precision of underlying asset values, we are able to not only verify a link between liquidity and required returns but we also are able to accurately quantify these gains.... our estimates of wealth creation jump to around 23% when comparing exchange traded claims to nontrading ones.” [“The Value of Liquidity,” L. Benveniste, D. Cappozza, and P. Seguin, Real Estate Economics, vol. 29(4), p. 656.]
Textbooks and appraisal texts also verify it....

• “Securities that cannot be converted so quickly and cheaply into cash need to offer relatively high yields.” [R. Brealey and S. Myers, *Principles of Corporate Finance*, 8th ed., p. 827.]

• “The rate of return on an investment combines a safe rate with a premium to compensate the investor for risk, the illiquidity of invested capital and management involvement.” [*The Appraisal of Real Estate*, 13th ed., p. 464.]
Appraisal texts require adjustment:

- 13th Edition Appraisal of Real Estate
  - “If there are differences between a comparable property and the subject property that could affect the overall capitalization rate concluded, the appraiser must account for these differences.”

- Securities are dramatically more liquid than property
  - Investors require lower returns for property than can be quickly sold at low expense
Flotation Costs, Size Premiums = Minimum Adjustments

- Including flotation costs for the debt and equity returns is only a partial adjustment for the illiquidity of property
  - Flotation costs represent the cost to sell securities, not property
  - Property is much less liquid than securities
    - Often months to sell
    - Much higher expense levels

- Size premiums are only a partial adjustment
  - Size premium represent difference in liquidity between more and less liquid securities
  - Property much less liquid than small capitalization stocks
Size Effect

Average Monthly Trading Volume vs Market Capitalization

- Monthly trading volume/outstanding shares
- Market Capitalization ($millions)

The scatter plot shows the relationship between average monthly trading volume and market capitalization.
### Size Effect

![Bid-Ask Spread vs Market Capitalization](chart.png)

<table>
<thead>
<tr>
<th>Size</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1%</td>
<td>0%</td>
</tr>
<tr>
<td>1%</td>
<td>2%</td>
</tr>
<tr>
<td>3%</td>
<td>4%</td>
</tr>
<tr>
<td>4%</td>
<td>5%</td>
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</table>

<table>
<thead>
<tr>
<th>Bid</th>
<th>Ask</th>
<th>Spread</th>
<th>Market Capitalization (Millions)</th>
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</thead>
<tbody>
<tr>
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<tr>
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</tr>
</tbody>
</table>
Summary

- Be aware of explicit and implicit assumptions of formulas used
- Discount rate must match cash flows
  - Nominal rate with nominal cash flows
  - Real rate with Real cash flows
  - After-tax rate with after-tax cash flows
  - Pre-tax rate with pre-tax cash flow
- Use correct growth rates in cash flows and discount rate (DGM)
  - Cash flows should reflect only existing assets
  - DGM must capture total return expected/required by investor
- If discount rate extracted from stocks and bonds, must be adjusted to reflect illiquidity of property