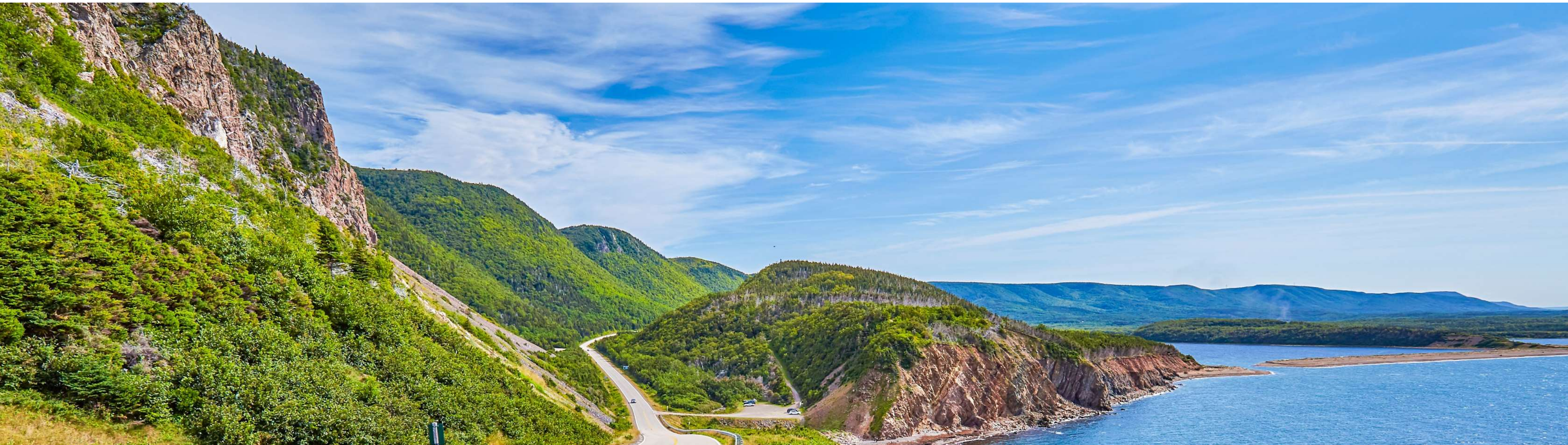

Wichita Property Tax Conference 2021
Valuation & Assessment Topics for Complex Industries

Innovation in Cost of Capital Over the Last 50 Years

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July 28, 2021

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- He has directed valuations of businesses, interests in businesses, intellectual property, intangible assets, real estate property and machinery and equipment. Roger has testified in court as an expert witness on matters of solvency, the value of closely held businesses and business interests, valuation and amortization of intangible assets and other valuation issues. His testimony in U.S. District Court was referenced in the U.S. Supreme Court opinion decided in his client's favor in the landmark Newark Morning Ledger case.
- Roger is co-author with Shannon Pratt of *Cost of Capital: Applications and Examples* 5th ed. (Wiley, 2014); co-author with Shannon Pratt of *The Lawyer's Guide to Cost of Capital* (ABA, 2014); and co-author of the source of cost of capital data, the Duff & Phelps Cost of Capital Navigator digital platform and the annual Duff & Phelps *Valuation Handbook* series; and contributing author to the upcoming Shannon Pratt's *Valuing a Business – The Analysis and Appraisal of Closely Held Companies* 6th ed. (McGraw-Hill, expected publication date 2021).
- Recent Papers:
  - "Comparing Growth Rates Used in Discounted Cash Flow Valuations," *Business Valuation Review*, 40 (1) (2021)
  - "Total Beta- Where Does it Fit in Valuation Theory," with Anas Aboulamer, Ph.D., *Business Valuation Review*, 39 (1) (2020)
  - "Size Premia are Alive and Kicking in Small-Co Stock Pricing," with Anas Aboulamer, Ph.D., *Law360*, October 8, 2020.
  - "Two Recent Articles addressing Firm Quality and its Impact on the Size Effect" (with Anas Aboulamer, PhD), *Business Valuation Update* (May 2019)
  - "The Size Effect Continues to be Relevant When Estimating the Cost of Capital," *Business Valuation Review*, 37(3) (2018)
  - "The Size Effect- It is Still Relevant," *Business Valuation Review*, Vol 35(2) (2016)
- Roger lectures often for professional organizations.

Innovation in Cost of Capital Over the Last 50 Years

- Innovations:
  - Theory of Cost of Capital
    - Financial analysts view
    - Academic view
    - Practitioner view
  - Data quality and availability
  - Technology available to the analyst
  - Acceptance of Evolving Valuation Theory

Innovation in Cost of Capital Over the Last 50 Years

Theory of Cost of Capital

- Financial analysts view
- Academic view
- Practitioner view

Innovation in Theory of Cost of Capital

Guidance from the view of security analyst:

- ***Security Analysis:*** *Principles and Techniques* 4th ed. (McGraw-Hill, 1962)

  by: Benjamin Graham, David L. Dodd, Sidney Cottle

  with the collaboration of Charles Tatham
- "The most important single factor determining a stock's value is now held to be the indicated average future earning power, i.e., the estimated average earnings for a future span of years. Intrinsic value (that value that is justified by the facts, e.g., assets, earnings, dividends, definitive prospects, including the factor of management) would then be found by first forecasting this earning power and then multiplying that prediction by an appropriate 'capitalization factor.'"
- An investment, "upon thorough analysis, promises safety of principal and a satisfactory return. Opportunities not meeting these requirements are speculative."
- On "growth stocks: "By definition, a "growth stock" has increased its per-share earnings at materially above the average rate of other issues and is expected to do so for many years in the future."

Innovation in Theory of Cost of Capital (cont'd)

- "In our view, security analysts as a whole cannot estimate the future earnings pattern of one or more such growth stocks with sufficient accuracy to provide a firm basis for valuation in the majority of cases. The inherent uncertainty about the rate and duration of future growth makes the intrinsic value of such issues at least partially speculative-in spite of the strength and quality of the business. Our concept of 'investment value' requires that the uncertainty as to future growth be reflected in modest projections and capitalization rates."
- On valuing growth stocks, Graham, Dodd and Cottle:
  - Adopt the "present value" technique which appears to be the basic principle in all major stock valuation models developed in the post World War II period
  - Place limitations on growth projections > maximum growth period which they consider in any of their techniques is ten years. In one method, the limit is reduced to seven years
  - Assume a single discount rate of 7.5 percent for all companies no matter whether high, medium, or low quality
  - Do not follow other writers in capitalizing dividends. Dividends, they argue, become almost meaningless for good growth stocks and they consider earnings as much more representative of such a firm's current and future income potential
-

Innovation in Theory of Cost of Capital (cont'd)

On valuing growth stocks, Graham Dodd and Cottle present two methods:

- **Method A:** This approach referred to as their preferred method, projects earnings growth for only the next seven years. A multiplier is applied to the average of the next seven years earnings, that is to the fourth year's earnings. The multiplier, of course, depends on the expected rate of growth for the next seven years, but will lie within the range of 13 to 20 because of the limits set by the authors on the growth rate from 3 1/2 percent to 20 percent. See the Exhibit on the next page.
- **Method B**: present two other formulas which yield similar results:

  B1. Value = 8.6T + 2.1 where T is the tenth-year compound amount of $1 of present earnings growing at any assumed rate.

  Assumptions: 60% payout, 7.5% discount rate, growth period ten-years

  Examples: 2.5% growth rate > T = $1.28 and the multiplier is (8.6 X 1.28) + 2.1 = 13.1 times current earnings per share;

  ten percent growth rate > multiplier of 24.4 times current earnings (8.6 X 2.59) + 2.1 = 24.4.

Innovation in Theory of Cost of Capital (cont'd)

B2: Value = current "normal" earnings X (8.5 + 2G), where G is the average annual growth expected for the next seven to ten years.

- "Normal" earnings are those as they would appear on a smoothed out earnings curve or "trend line".
- They arrived at this formula from the finding that a multiplier of 8.5 is appropriate for a company with zero expected growth, while a 2 1/2 percent growth rate calls for a multiplier of 13.5, and a 10 percent growth rate indicates a multiplier of 8.5 + (2 X 10) = 28.5 times as compared with 24.4 times in the first formula.

Remember: Cost of Equity = Capitalization Rate – growth or E/Value - growth

- These methods for valuing growth issues are somewhat conservative in recommending the use of short periods of anticipated growth and relatively low residual growth rates and multipliers.
- Graham, Dodd, and Cottle disregard any higher than "average" growth rate later than ten years hence.
- Unfortunately, Graham, Dodd, and Cottle do not advise the reader on how to select the proper growth rate. They do suggest that past trends definitely are an important factor to consider, but they should not be the sole factor.

Graham, Dodd and Cottle Multipliers

Expected Rate Of Growth (4 Years)	Multiplier Of Average (4th Year Earnings)	Multiplier Of Current Earnings
3.5%	13X	15X
5.0%	14X	17X
7.2%	15X	20X
10.0%	16X	23 1/2X
12.0%	17X	27X
14.3%	18X	31X
17.0%	19X	35 1/2X
20.0%	20X	41 1/2X

Innovation in Theory of Cost of Capital (cont'd)

- Graham, Dodd, and Cottle discuss on page 523:

"Practical Use of Common-stock Valuations. The purpose of formal valuations is, of course, to aid the analyst in his choice and recommendation of attractive common-stock purchases. Normally he will prefer an issue selling at a lower percentage of his derived value to one selling at a higher percentage. And normally he would compose an investment list of diversified issues giving the most "value" for the price paid-giving due consideration to the stability factor.

What appears to be the beginning of a new movement along these lines has its origins in a work done by H.M. Markowitz. He posits expected over-all return and price stability (the opposite of "variability" or "uncertainty") as two independent criteria for satisfactory investment. Presumably the investor must, to some degree, sacrifice return to stability and stability to return. Markowitz has developed an interesting computing technique for arriving at what he calls "efficient portfolios"- i.e., those that will provide a maximum return for a given accepted degree of uncertainty, or a maximum degree of uncertainty with a given desired (and expected) return.

Innovation in Theory of Cost of Capital (cont'd)

The analyst must be well aware of the two major limitations of this or a similar approach. The first limitation derives from the inherent unreliability of *all* investment decisions which are based largely on estimates of future earnings and on more or less arbitrary choices of the multiplier to be applied to these expectations….

The second limitation is structural and grows out of the inherent opposition between stock-market valuations and security-analysis valuations. The stock market tends to recurrent extremes, in its general bullishness and bearishness, in its marking-up of investment and speculative favorites and its marking down of unpopular issues. The analyst seeks for a middle ground in his view of common stocks in general, and he tends to narrow somewhat a huge spread which the market has established between the valuation rates for popular and unpopular issues. We recommend this attitude for the analyst…"

Innovation in Theory of Cost of Capital (cont'd)

- Nicholas Molodovsky made a comprehensive historical analysis of the S&P and Cowles Commission indexes and used them as the standard with which to compare all stocks. The figures of these stock indexes were used to develop basic historical parameters.
- Historical growth rate of dividends and earnings were found to be about 2.5 percent and the average yield for the period as evidenced by the "stock averages" were about 5 percent. The combining of the two figures resulted in a total effective yield per year of 7.5 percent, which Molodovosky consequently used as the discount rate in the present-value formula in studies through 1960.
- For later years, through 1963, the above parameters respectively changed to about 2.7 percent; 4.9 percent; and 7.8 percent respectively.
- Stressed that, in actual analytical practice, projections of future earnings trends of different stocks would have to be made for whatever varying periods might be specifically indicated.
- "The Many Aspects of Yields" contained the following statement: "It is clear that the nature of the industry to which a given company belongs - should in reality determine both the length of the period for which earnings are projected into the future, and also the delicate process of the "splicing" with an overall historical growth rate. Depending on each individual case, such a transition may take the form of mathematical curves with very different gradations of diminishing rates of growth."

Innovation in Theory of Cost of Capital (cont'd)

Guidance from the view of the academic:

Dividend Discount Model or Gordon Growth Model: Named for Professor Myron Gordon. Myron J. Gordon and Eli Shapiro, "Capital Equipment Analysis: The Required Rate of Profit," *Management Science* Vol. 3 (October 1956): 102–110, reprinted in *Management of Corporate Capital* (Glencoe, IL: Free Press of, 1959); Myron J. Gordon, *The Investment, Financing, and Valuation of the Corporation* (Homewood, IL: R. D. Irwin, 1962).

Value assuming perpetual growth:

$$PV = D_0 (1+g) / (k - g)$$

Rearranging, we get

$$k = D_0 (1+g) / PV + g$$

In valuing businesses, D_0 is typically defined in terms of NCF_0

Innovation in Theory of Cost of Capital (cont'd)

Value assuming tapering growth – the two-stage model: W. Scott Bauman, "Investment Returns and Present Values," *FAJ* (Nov-Dec 1969); Eugene F. Brigham and James L. Pappas, "Duration of Growth, Changes in Growth Rates, and Corporate Share Prices," *FAJ* (May-June 1966); Paul F. Wendt, "Current Growth Stock Valuation Models," *FAJ* (March-April 1965):

$$PV = \frac{NCF_1}{(1+k)} + \frac{NCF_2}{(1+k)^2} + \cdots + \frac{NCF_n}{(1+k)^n} + \frac{\frac{NCF_n(1+g)}{k-g}}{(1+k)^n}$$

- Tables were published matching growth rates, present values and discount rates to implement the two-stage model
  - Nicholas Molodovsky, Catherine May and Sherman Chottiner, "Common Stock Valuation," *FAJ* (1965);
  - Robert M. Soldofsky and James T. Murphy, *Growth Yields on Common Stock: Theory and Tables* (Iowa City: Bureau of Business and Economic Research, University of Iowa, 1961).
  - Discussed further below.

Innovation in Theory of Cost of Capital (cont'd)

- A unified price of risk, that we could call the market price of risk, was elusive until Harry Markowitz laid the ground for the biggest leap in what is now called Modern Financial Theory (MFT). Markowitz provided a framework that would combine investors' tolerance towards risk and the need for a unified price of risk. He proved that a rational investor will always hold the same mean-variance optimized portfolio where the return will be maximized for a unit of risk.
- Harry Markowitz chose variance and standard deviation as a measure of risk in his groundbreaking work that paved the path to the creation of what is now called the modern portfolio theory (MPT) and it became the norm in measuring risk. However, the major shortcoming of standard deviation is its treatments of positive and negative numbers. Standard deviation is a symmetrical risk measure which is not consistent with the way risk is viewed and defined. A symmetrical risk measure implies that large positive and negative movements are treated equally.

See: Harry Markowitz, "The Utility of Wealth." *Journal of Political Economy*, Vol, 60(2) (1952).

Innovation in Theory of Cost of Capital (cont’d)

- Markowitz himself later admitted that “semi-variance is the more plausible measure of risk” but decided on variance and covariance as risk measures because these measures were “cheaper” to calculate, given the computing power at the time, application of the formulas for portfolio selection were straightforward, and variance and covariance were familiar concepts. Markowitz found that other measures of portfolio risk resulted in “better” portfolios with lower risk given an expected return.

See: 151-158; Harry Markowitz, *Portfolio selection: Efficient Diversification of Investment* (London: Chapman & Hall, 1959): 193; Shaun A. Bond and Stephen E. Satchell. "Statistical Properties of the Sample Semi-variance." *Applied Mathematical Finance*, Vol 9 (4) (2002): 219-239.

Innovation in Theory of Cost of Capital (cont'd)

- Treynor, Sharpe, Lintner, and Mossin[1] extended and simplified the Markowitz model by introducing assumptions of (1) complete agreement among investors on the joint probability distribution of asset returns from time $t - 1$ to time t (and its *true* probability distribution) and (2) unrestricted risk-free borrowing and lending. The resulting model, CAPM, and theory defining expected behavior by investors in accordance with the model can be thought of as "capital market theory of the two-parameter model."

See: Jack L. Treynor, "Market Value, Time, and Risk," August 8, 1961 (revised April 29, 2015, with minor edits by Craig William French), available at SSRN: https://ssrn.com/abstract=2600356 and "Toward a Theory of Market Value of Risky Assets," a reprint may be found in French, Craig W., Jack Treynor's 'Toward a Theory of Market Value of Risky Assets' (December 28, 2002), available at SSRN: http://ssrn.com/abstract=628187. A more complete description of the development of the Treynor CAPM may be found in French, Craig W., "The Treynor Capital Asset Pricing Model," *Journal of Investment Management*, Vol. 1 (2) (2003): 60-72, available at SSRN: http://ssrn.com/abstract=447580 ;

William F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance* (September 1964): 425–442.;

John Lintner, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics* (February 1965): 13–37;

Jan Mossin, "Equilibrium in a Capital Asset Market," *Econometrica,* Vol. 35 (October 1966): 768-783.

Innovation in Theory of Cost of Capital (cont'd)

- CAPM simplified Markowitz's measures of risk such that the only risk measure that mattered was the market beta. Beta measures expected market risk (termed *systematic risk* in the CAPM). Beta is a function of the *expected* relationship between the return on an individual security (or portfolio of securities) and the return on the market. This remaining common or systematic variability among all assets is due to changes in the economic, psychological and political environment that affect all assets.
- The theory around the CAPM depend on many simplifying assumptions: the absence of transaction costs and taxes, all investors are risk averse and rational with identical investment horizons and expectations and markets are perfectly liquid. Under these assumptions and since all investors are mean-variance optimizers and they invest in the best portfolio (i.e.: the market), the price of risk of any investment will ***only*** depend on its covariance with the market portfolio—CAPM's beta.

Innovation in Theory of Cost of Capital (cont'd)

Adjusting Betas for Differences in Risk due to Differences in Leverage:

- Robert S. **Hamada**, "The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks," *Journal of Finance* (May 1972): 435–452.
- James A. **Miles** and John R. **Ezzell**, "The Weighted Average Cost of Capital, Perfect Capital Markets and Project Life: A Clarification," *Journal of Financial and Quantitative Analysis* (1980): 719–730.
- R. S. **Harris** and J. J. **Pringle**, "Risk-Adjusted Discount Rates—Extensions from the Average Risk Case," *Journal of Financial Research* (Fall 1985): 237–244.
- "**Practitioners Method**," Tim Ogier, John Rugman, and Lucinda Spicer, *The Real Cost of Capital* (New York: Financial Times Prentice-Hall, 2004): 49.
- Pablo **Fernandez**, "Levered and Unlevered Beta," Working paper (April 20, 2006) available at http://ssrn.com/abstract=303170.

Innovation in Theory of Cost of Capital (cont'd)

- Issues with CAPM
- Despite its popularity, issues continue to be discussed – high-beta stocks result in low-returns; low-beta stocks result in high-returns
  - Issues: Are the assumptions underlying CAPM violated in the market?

    Do we not know how to estimate beta?

    Does the market really price risk using many more factors than simply beta?
  - Major issue: Returns are not normally distributed- we observe heavier tails

    Returns seem to be better measured by the Cauchy distribution – but the Cauchy distribution has no mean

    Consequently, the central limit theorem does not hold which negates mean-variance finance as we know it (i.e., negates CAPM, Arbitrage Pricing Theory (APT), Black-Scholes Option Pricing Model)

See: D. Harris, "Why Heavy Tails?" *Southwestern Economic Review*, 41, 1 (Spring 2014):127-151 and Harris "A Population Test of Distribution Assumptions in Mean-Variance Models for the years 1925-2013", working paper, August 29, 2015

Innovation in Theory of Cost of Capital (cont'd)

- Other Models Beyond CAPM
- Because of the poor empirical record of pure CAPM, Fama and French (F-F) conducted empirical research which confirmed that other factors besides market beta and firm size (as measured by market capitalization), add to the explanation of realized returns.
- F-F found that pure CAPM cost of equity estimates for high-beta stocks were too high and estimates for low-beta stocks were too low (relative to realized returns). The pure CAPM cost of equity estimates for high book-value-to-market-value stocks (so-called *value stocks*) were too low and estimates for low book-value-to-market-value stocks (so-called *growth stocks*) were too high (relative to realized returns).
- The implication of their research is that if market betas only do not explain expected returns, then the market portfolio, *M*, is not efficient, and pure CAPM has potentially fatal problems. As a result, F-F introduced an empirically driven model to estimate cost of equity capital that is not dependent on beta alone.

See: Eugene Fama and Kenneth French, "The CAPM: Theory and Evidence," *Journal of Economic Perspectives* (January 2004): 25–46.

Innovation in Theory of Cost of Capital (cont'd)

Fama-French Models: F-F (1993) first introduced a three-factor model:

$$(R_{it} – RF_t) = a_i + b_i(RM_t – RF_t) + s_i SMB_t + h_i HML_t + e_{it}.$$

R_{it} = return on security or portfolio *i* for period *t*, RF_t = risk-free return, RM_t = return on the value-weight market portfolio, SMB_t = return on a diversified portfolio of small stocks minus the return on a diversified portfolio of big stocks, HML_t = difference between the returns on diversified portfolios of high and low *B/M* stocks, and e_{it} = zero-mean residual.

F-F now add profitability and investment factors:

$$(R_{it} – RF_t) = a_i + b_i(RM_t – RF_t) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}.$$

RMW_t = difference between the returns on diversified portfolios of stocks with robust and weak profitability;
CMA_t = difference between the returns on diversified portfolios of the stocks of low (conservative) and high (aggressive) investment firms.

If the exposures to the five factors, *bi*, *si*, *hi*, *ri*, and *ci*, capture all variation in expected returns, the intercept a_i is zero for all securities and portfolios *i*.

See: Eugene Fama and Kenneth French, "The Cross-Section of Expected Stock Returns, " *Journal of Finance* (June 1992): 427–486.; "A Five-Factor Asset Pricing Model," *Journal of Financial Economics* 116 (1) April 2015:1-22.

- Factors available on Duff & Phelps Cost of Capital Navigator U.S. Benchmarking Module and on Kenneth French website.

Innovation in Theory of Cost of Capital (cont'd)

Arbitrage Pricing Theory

- Multi-variable model

See, Richard Roll and Stephen A. Ross, "An Empirical Investigation of Arbitrage Pricing Theory, " *Journal of Finance* (December 1980): 1073–1103; Nai-fu Chen, "Some Empirical Tests of Arbitrage Pricing, " *Journal of Finance* (December 1983): 1393–1414; Nai-fu Chen, Richard Roll, and Stephen A. Ross, "Economic Forces and the Stock Market: Testing the APT and Alternative Pricing Theories, " *Journal of Business* Vol. 59 (1986): 383–403; Roger G. Ibbotson and Gary P. Brinson, *Investment Markets* (New York: McGraw-Hill, 1987): 32. For a more extensive discussion of APT, see Frank K. Reilly, *Investment Analysis and Portfolio Management*, 8th ed. (Fort Worth, TX: Dryden Press, 2005).

- More complicated than CAPM- User reliant on data supplier for inputs. One such supplier was *BIRR Risk and Returns Analyzer*.

See: "A Practitioner's Guide to Arbitrage Pricing Theory, " in *A Practitioner's Guide to Factor Models* (Research Foundation of the Institute of Chartered Financial Analysts, 1994); Edwin Burmeister, "Using Macroeconomic Factors to Control Portfolio Risk," Working paper, Duke University (March 9, 2003).

- Has fallen out of favor because of lack of available data supplier.

Innovation in Theory of Cost of Capital (cont'd)

Implied Cost of Equity Models:

- Single-Stage (DCF) model:

$$k_e = \frac{NCF_0(1+g)}{PV} + g$$

  - Single-stage DCF model often is used in regulated utility rate hearings to estimate a utility's cost of equity capital. The dividend yield is assumed to be an appropriate estimate of the first input, cash flow yield.
- Multistage (DCF) model:
  - Soldofsky and Murphy published tables based on two-stage model in 1963 (discussed below)
  - Popularized when Ibbotson Associates published theory and data by industry in the *Cost of Capital Yearbook* beginning in October 1993.

$$PV = \sum_{n=1}^{5} \frac{\left[NCF_0(1+g_1)^n\right]}{(1+k_e)^n} + \sum_{n=6}^{10} \frac{\left[NCF_5(1+g_2)^{n-5}\right]}{(1+k_e)^n} + \frac{\frac{NCF_{10}(1+g_3)}{k_e - g_3}}{(1+k_e)^{10}}$$

  - Multistage model is now considered as one input in STB ratemaking.
  - Estimates published for multiple industries in the Cost of Capital Navigator U.S. Benchmarking Module.

Innovation in Theory of Cost of Capital (cont'd)

Size Effect

- Roger Ibbotson and Rex Sinquefeld began publishing summaries of data that demonstrated the size effect (later incorporated in the annual *Stocks, Bonds, Bills and Inflation* (*SBBI*®)) based on analyzing the CRSP data with market cap as the measure of size.
- One of the characteristics observed was that large market capitalization ("large-cap") companies versus small market capitalization ("small-cap") companies. They divided the universe of publicly traded U.S. companies into 10 "deciles" (i.e., portfolios), with the largest-cap companies in Decile 1 and the smallest-cap companies in Decile 10. What they found was that the returns for small-cap companies were greater than the returns for larger-cap companies.
- In 1981, Banz examined the returns of New York Stock Exchange (NYSE) small-cap stocks (as measured by market capitalization) compared to the returns of NYSE large-cap stocks over the period 1926–1975.

See: Rolf W. Banz,"The Relationship between Return and Market Value of Common Stocks." *Journal of Financial Economics* (March 1981): 3–18. This paper is often cited as the first comprehensive study of the size effect.

Innovation in Theory of Cost of Capital (cont'd)

- Originally a "small stock premium" was calculated as the simple difference in small company returns versus large company returns. However, making a size adjustment based on the simple difference in small company returns versus large company returns is problematic because

  …in doing so one assumes that the company being valued has the same systematic risk (or beta) as the portfolio of small stocks used in the calculation of the size premium.

- In other words, a size premium needs to be adjusted to "control for", or *remove*, the portion of excess return that is attributable to beta, leaving only the size effect's contribution to excess return.

See: 2012 SBBI Valuation Yearbook: 28.

Innovation in Theory of Cost of Capital (cont'd)

- Size Effect not without its critics:
- Bias may be introduced when ranking companies by market value because a company's market capitalization may be affected by characteristics of the company other than size. In other words, some companies might be small because they are risky (high discount rate), rather than risky because they are small (low market capitalization).

See: Jonathan Berk, "A Critique of Size Related Anomalies," *Review of Financial Studies*, Vol. 8 (2) (1995).

- Others say the size effect is due to data anomalies or has disappeared at times.

See: Clifford S. Ang, in "The Absence of a Size Effect Relevant to the Cost of Equity." *Business Valuation Review* 37, no. 3 (2018): 87-92.

- But recent studies support the premise of a size effect once you control for quality. Discussed below.

See: Clifford S. Asness, Andrea Frazzini, Ronen Israel, Tobias J. Moskowitz, and Lasse Heje Pedersen, "Size Matters, If You Control Your Junk," *Journal of Financial Economics* 129 (2018): 479-509; The quality measure is discussed in a companion paper, "Quality minus Junk" by Asness, Frazzini & Pedersen in *Review of Accounting Studies* (2018); Roger J. Grabowski, "The Size Effect Continues to be Relevant When Estimating the Cost of Capital,' *Business Valuation Review* 37(3) (2018); Roger J. Grabowski and Anas Aboulamer, Ph.D., "Two Recent Articles addressing Firm Quality and its Impact on the Size Effect," *Business Valuation Update*, May 2019.

Innovation in Theory of Cost of Capital (cont'd)

- Summary
  - Evolution of theories in part driven by limited computing capabilities and limited data
  - Early methodologies were limited a generalized build-up methods.
  - CAPM evolved because it presented the relationship of risk and return in a manner students could understand.
  - But limited computing capabilities and limited data availability made its acceptance by practitioners difficult.
  - Would CAPM have evolved to the most widely used method for estimating cost of equity if the computing power of today's microcomputers been available to researchers?
    - If we had the computing power to look at more complex relationships between risk and return, would beta be the risk measure most often cited?
  - As we will discuss later, the evolution of theory comes first, acceptance comes much later.

Data quality and availability

Advances in Data Quality and Availability

Returns on Common Stocks

- Estimates of realized returns on common stock were based on studies of returns on indexes: most notably the Cowles Commission and Molodovsky's work (1871-1973).
- The creation of the CRSP databases at the University of Chicago in the early 1960s was a big advance in research in security prices. The CRSP database represents market value (stock price times the number of shares) and return data (dividends and change in stock price) going back to 1926.
- Prior to the creation of the CRSP databases, one literally had to gather data from old newspapers to do a retrospective analyses by company. However, possibly the most notable reason that the establishment of the CRSP databases was so critical was that it enabled researchers to look at stocks with different characteristics and analyze how their returns differed. With this capability we began to better understand the drivers of stock returns.
- But the financial data on individual companies was first only available in print form. S&P *Compustat* was the first computerized database of company level financial data.
- It was not until the 2000's that CRSP and S&P offered a combined data set.

Advances in Data Quality and Availability (cont'd)

- The availability of CRSP data spawned many studies not possible previously.
- Lawrence Fisher and James H. Lorie computed returns from portfolios of all common stocks listed on NYSE for 1926-1965.

See: "Rates of Return on Investments in Common Stock: The Year-by-Year Record, 1926-1965," *Journal of Business* (July 1968).

- Roger Ibbotson and Rex Sinquefeld introduced investors to detailed analyses of CRSP data. The presented historical data and simulated expected returns on bonds and stocks.

See: "Stocks, Bonds, Bills and Inflation: Year-by-Year Historical Returns (1926-1974)," *Journal of Business* (Jan 1976); "Stocks, Bonds, Bills and Inflation: Simulations of the Future (1976-2000)", *Journal of Business* (July 1976).

These papers were followed by a series on monographs published by the Financial Analysts Research Foundation in 1977, 1979 and 1982.

See: *Stocks, Bonds, Bills and Inflation: The Past (1926-1976) and the Future (1977-2000)*; *Stocks, Bonds, Bills and Inflation: The Past (1926-1978); Stocks, Bonds, Bills and Inflation: The Past and the Future – 1982 Edition*

See some excerpts on the following slides from the 1982 monograph.

- In 1983 Ibbotson Associates published its first *SBBI Yearbook*.

SBBI Monograph- 1982 edition

STOCKS, BONDS, BILLS, and INFLATION:
THE PAST and THE FUTURE

1982 Edition

MONOGRAPH NUMBER 15

by

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and

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Foreword By Laurence B. Siegel

THE
FINANCIAL
ANALYSTS
RESEARCH
FOUNDATION

SBBI Monograph- 1982 edition (cont'd)

III. COMPONENTS OF SECURITY RETURNS

Historical data suggest that investors have been rewarded for taking risks and that returns have been related to inflation rates. We now seek to uncover the risk/return and the real/nominal relationships in the historical data.

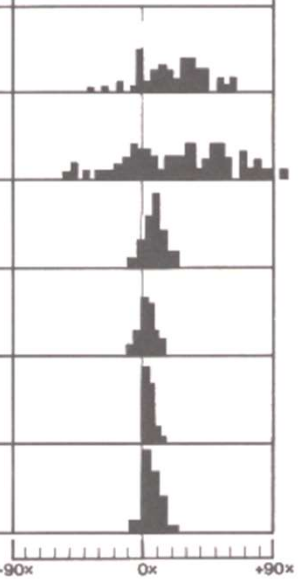
From the six basic asset classes—common stocks, small stocks, long-term government bonds, long-term corporate bonds, U.S. Treasury bills (T-bills), and consumer goods (inflation)—we derive additional series representing the component or elemental parts of the asset returns. We present nine of these derived series:

	Title	Derivation
(1)	Inflation-adjusted T-bill returns (Real riskless rate of return)	(T-bills) — (Inflation)
(2)	Bond maturity premiums	(Long-term government bonds) — (T-bills)
(3)	Bond default premiums	(Long-term corporate bonds) — (Long-term government bonds)
(4)	Inflation-adjusted government bond returns (1+2)	(Long-term government bonds) — (Inflation)
(5)	Inflation-adjusted corporate bond returns (1+2+3)	(Long-term corporate bonds) — (Inflation)
(6)	Equity risk premiums	(Common stocks) — (T-bills)
(7)	Inflation-adjusted common stock returns (1+6)	(Common stocks) — (Inflation)
(8)	Small stock premiums	(Small stocks) — (Common stocks)
(9)	Inflation-adjusted small stock returns (1+6+8)	(Small stocks) — (Inflation)

SBBI Monograph- 1982 edition (cont'd)

EXHIBIT 3
BASIC SERIES: TOTAL ANNUAL RETURNS
(1926-1981)

SERIES	GEOMETRIC MEAN	ARITHMETIC MEAN	STANDARD DEVIATION	DISTRIBUTION
COMMON STOCKS	9.1%	11.4%	21.9%	
SMALL STOCKS	12.1	18.1	37.3	
LONG TERM CORPORATE BONDS	3.6	3.7	5.6	
LONG TERM GOVERNMENT BONDS	3.0	3.1	5.7	
U.S. TREASURY BILLS	3.0	3.1	3.1	
INFLATION	3.0	3.1	5.1	

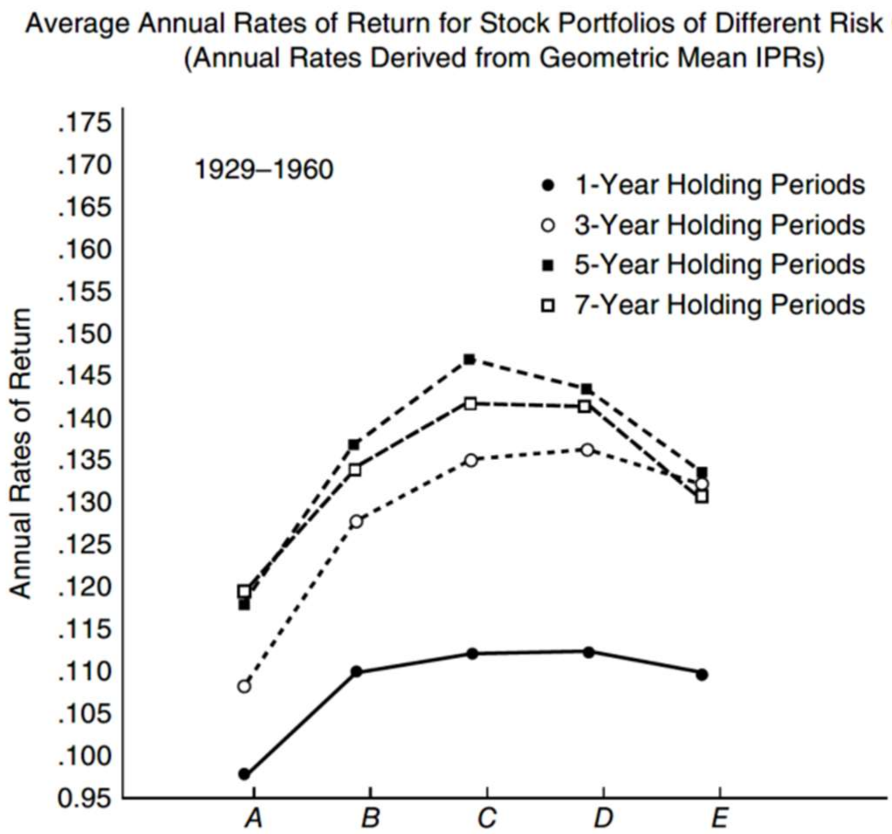


Advances in Data Quality and Availability (cont'd)

- The availability of CRSP data spawned many studies not possible previously.
- For example, Shannon Pratt graded all stocks according to the historical variation of past prices. The greater the relative variation, the greater the risk of the stock and the lower the grade assigned to it. He calculated the average return for all stocks in a particular grading category for various holding periods.
- Pratt found that generally the lower the grade of stock, the greater the risk, the higher the average realized return. (see graphs on following pages).

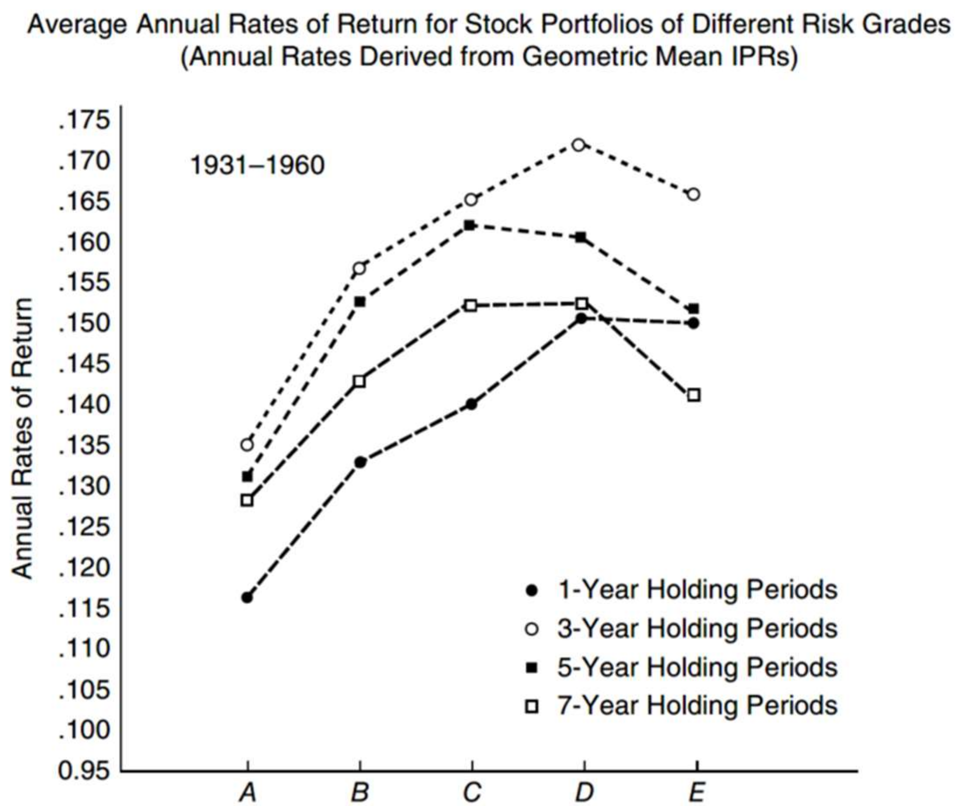
See: "Relationship Between Risk and Return for Common Stocks," Ph.D. Dissertation, Indiana University, 1966, reproduced in *Frontiers of Investment Analysis* (1971)

RELATIONSHIP BETWEEN RISK AND THE RATE OF RETURN FOR COMMON STOCKS (Exhibit 13.1, Pratt & Grabowski, *Cost of Capital* 5th ed.)



Source: E. Bruce Fredrikson, *Frontiers of Investment Analysis*, 2nd ed. (Scranton, PA: Intext Educational Publishers, 1971), 345. Used with permission.

RELATIONSHIP BETWEEN RISK AND THE RATE OF RETURN FOR COMMON STOCKS (Exhibit 13.1, Pratt & Grabowski, *Cost of Capital* 5th ed.)



Source: E. Bruce Fredrikson, *Frontiers of Investment Analysis*, 2nd ed. (Scranton, PA: Intext Educational Publishers, 1971), 345. Used with permission.

Advances in Data Quality and Availability (cont'd)

- *Value Line* offered (and still does offer) estimates of expected returns by individual company using an implied model.
- CRSP data resulted in estimates of Equity Risk Premium: stock market overall return measured yearly minus bond yield.
  - Series that were then published in Ibbotson Associates *SBBI Stock, Bonds, Bills and Inflation Yearbooks*
    - Long-term, "historical" (i.e., realized) ERP from 1926 to most current year-end
    - Long-term, "supply-side" ERP (started later)
  - Now published in the Duff & Phelps Cost of Capital Navigator U.S. Cost of Capital Module.

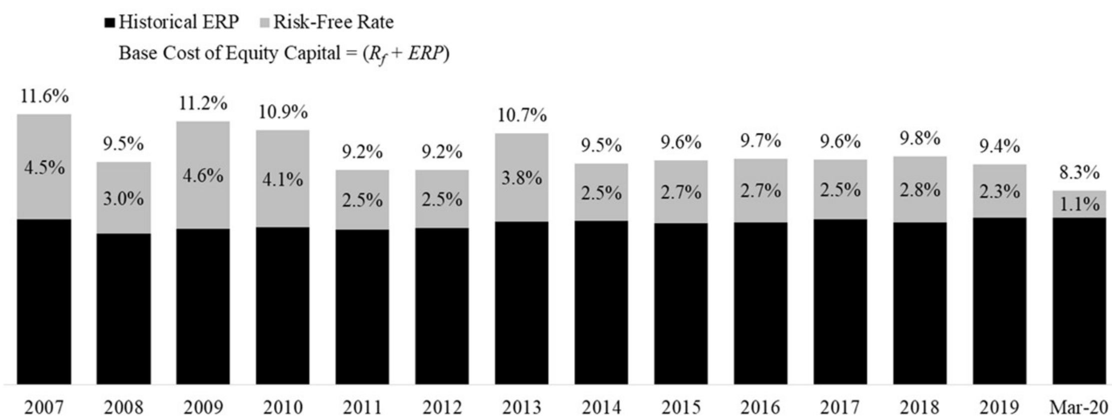
See: Roger G. Ibbotson and Peng Chen, "Long-Run Stock Returns: Participating in the Real Economy," *Financial Analysts Journal*, Vol. 59 (1) (January/February 2003), pp. 88–98.

- CRSP data resulted in estimates of Size Premia (see discussion below)

Problem with relying on Spot Rate and unadjusted "Historical" ERP During Crises

Spot 20-year U.S. Treasury Yield in Conjunction with Unadjusted "Historical"

Equity Risk Premium



Source of underlying data: Morningstar. Exhibit 11-4 is constructed using the long-horizon "historical" ERP, and the SBBI® long-term U.S. government bond yield series, as previously published in the former 1999–2006 Ibbotson Associates *SBBI Valuation Yearbook*, the former 2008–2013 Morningstar *SBBI® Valuation Yearbook*, the Duff & Phelps 2014–2017 *Valuation Handbook – U.S. Guide to Cost of Capital*, and, now (starting in 2018) the online Duff & Phelps Cost of Capital Navigator. Calculations performed by Duff & Phelps, LLC.

Conditional ERP

Conditional ERP estimates–

ERP is cyclical and

Conditional ERP represents the ERP at a specific point in the cycle.

Reported in the Duff & Phelps Cost of Capital Navigator:

Duff & Phelps Recommended ERP as of **March 25, 2020** is 6.0% (measured against a "normalized" risk-free rate = 3.0%; total base discount rate = 9.0%) (an increase from the previous 5%, measured against a "normalized" risk-free rate = 3.0%; total base discount rate =8.0%) in response to evidence that suggested a heightened level of risk in financial markets and deteriorating economic conditions.

Duff & Phelps changed its Recommended ERP as of early December 2020 to 5.5% (measured against a "normalized risk-free rate = 2.5%; total base discount rate = 8.0%) due to the increasing stock market prices (driven in large part by the expected approvals of Covid vaccines), continuing commitment by the Federal Reserve Bank to maintain low interest rates, movement upwards in expected growth in GDP for 2021.

Conditional ERP (cont'd)

Professor Aswath Damodaran calculates implied ERP monthly estimates for the S&P 500 and publishes his estimates on his website monthly.

- Damodaran estimates an implied ERP by first solving for the discount rate that equates the current S&P 500 index level with his estimates of cash distributions (dividends and stock buybacks) in future years. Uses yield on 10-year U.S. government bonds as long-term growth rate.
- Damodaran then subtracts the current yield on 10-year U.S. government bonds to arrive at the implied ERP.

- These Implied ERP estimates are equivalent to geometric average ERPs in terms of a 10-year U.S. Government bond.

- Duff & Phelps (1) converts Damodaran's geometric ERP estimates to an equivalent estimate in terms of normalized yields on 20-year U.S. government bonds and then (2) converts the geometric ERP estimates to their arithmetic average equivalents.

Advances in Data Quality and Availability (cont'd)

Inputs into models

- Beta: Practitioners that first started applying CAPM needed to calculate betas for individual companies.
- Services became available that published individual company beta estimates (e.g., the Ibbotson Associates *Beta Book* and S&P *Compustat*)
- Now available on many websites- but typically only one type of beta estimate (ordinary least squares regression of returns over a look-back period). Bloomberg offers both OLS betas and Blume adjusted beta.
- Other beta estimates:
  - **Blume Adjustment:** multiplying the OLS beta estimate by two thirds and adding one third to mitigate the bias in the OLS estimate.
  - **Vasicek shrinkage Adjustment:** weighting the OLS beta estimate by the industry beta
  - **Lag effect (Sum Beta):** using a lag effect beta estimate for small companies.
- Use of other beta estimates typically require user to calculate the beta estimates.
- Industry estimates of beta are available in the U.S. Industry Benchmarking Module.

Advances in Data Quality and Availability (cont'd)

Size Effect

- The CRSP Deciles Size Study was previously published in the Ibbotson Associates *Stocks, Bonds, Bills, and Inflation (SBBI) Valuation Yearbook* from 1999–2006, in the *Morningstar/Ibbotson® Stocks, Bonds, Bills, and Inflation (SBBI) Valuation Yearbook* from 2007–2013, in the *Duff & Phelps Valuation Handbook – U.S. Guide to Cost of Capital* from 2014–2017 and is available in the online D&P/Kroll Cost of Capital Navigator U.S. Cost of Capital Module.
- Duff & Phelps uses CRSP return data and S&P's *Compustat* database to develop the size premia (and other premia) published in the *Risk Premium Report*.
- The most prominent difference between the two studies is that the *CRSP Deciles Size Study* measures size solely by market value of equity ("market capitalization", or simply "market cap"), while the *Risk Premium Report* measures size by market capitalization, *plus* seven additional measures of size:

Market capitalization   Market value of invested capital (MVIC)   Book value of equity

5-year average net income   Total assets   5-year average EBITDA

Sales   Number of employee

Advances in Data Quality and Availability (cont'd)

- Risk Premium Report grew out of research by Grabowski and King.

See: Roger J. Grabowski and David King, "New Evidence on Size Effects and Equity Returns", *Business Valuation Review* (September 1996): 103-115; Roger J. Grabowski and David King, "New Evidence on Equity Returns and Company Risk", *Business Valuation Review* (September 1999, revised March 2000): 32-43.

- The Risk Premium Report was published as the Price Waterhouse Risk Premium Reports and PricewaterhouseCoopers Risk Premium Reports (for years before 2002), as the Standard & Poor's Corporate Value Consulting Risk Premium Report (from 2002–2004), the Duff & Phelps Risk Premium Report (from 2005–2013), and in the *Duff & Phelps Valuation Handbook – U.S. Guide to Cost of Capital* (from 2014–2017), and since 2018 has been available exclusively in the online D&P/Kroll Cost of Capital Navigator U.S. Cost of Capital Module.
- Size premia are more complicated to apply than many practitioners appreciate and is misunderstood.
  - Size premia strong after controlling for "junk".
  - Size premia are correlated with fundamental risks of companies.
  - Size premia correlated with liquidity.

Excerpt from Risk Premium Report for MCAPM

Companies Ranked by Market Value of Equity

Risk Premium Report – Size Study: Size Premia for MCAPM as of December 31, 2019

Portfolio	Portfolio Breakpoint ($ millions)	Avg. Debt/ MVIC	Avg. Op. Margin	Avg. CV (ROE)	Std. Dev. Of Returns	RPs
Portfolio 1	$185,926.20 and Up	14.48%	17.13%	18.48%	16%	-0.84%
2	$56,958.91 – $185,926.20	18.79%	13.90%	22.42%	16.88%	0.49%
3	$35,408.55 – $56,958.91	20.86%	13.53%	21.95%	16.24%	0.98%
4	$24,894.78 – $35,408.55	22.70%	13.42%	23.32%	16.85%	1.34%
5	$18,620.57 – $24,894.78	23.29%	12.48%	22.24%	17.29%	1.61%
⋮	⋮	⋮	⋮	⋮	⋮	⋮
20	$1,320.18 – $1,577.82	23.90%	9.13%	34.97%	23.55%	4.11%
21	$1,079.85 – $1,320.18	23.44%	8.83%	37.38%	22.34%	4.26%
22	$834,68 – $1,079.85	24.01%	8.60%	39.41%	23.38%	4.48%
23	$590.93 – $834.68	23.78%	8.07%	42.72%	23.58%	4.74%
24	$305.72 – $590.93	24.45%	7.86%	45.66%	25.49%	5.16%
Portfolio 25	Up to $305.72	26.76%	6.24%	64.43%	34.36%	6.20%

Sources of underlying data: (i) CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. CRSP® is a registered trademark and service mark of Center for Research in Security Prices, LLC and has been licensed for use by Duff & Phelps, A Kroll Business ("D&P/Kroll"). The D&P/Kroll publications and services are not sponsored, sold or promoted by CRSP®, its affiliates or its parent company. To learn more about CRSP, visit www.crsp.com. (ii) Morningstar, Inc. Used with permission. All rights reserved. Calculations performed by D&P/Kroll.

The Size Effect

Misunderstandings Have Continued

- The Size Effect is cyclical
  - Critics conclude that because Large company stocks outperform Small company stocks during certain periods, the Size Effect is not valid
  - But if Small company stocks always outperformed Large company stocks, they would not be riskier- the volatility of returns shows that they are riskier

- Other criticisms focus on the data: Seasonality (i.e., January effect), Bid/Ask bounce bias, delisting bias, transaction costs.
  - These criticisms have each been addressed in Chapter 15A (pp.363-371) in Shannon Pratt and Roger J. Grabowski, *Cost of Capital: Applications and Examples* 5th ed. (John Wiley & Sons, Inc., 2014)

- Some critics assert that the size effect has disappeared in recent years.

Size Factor Strengthens After Controlling for Quality

- The debate about the size effect is still on-going among academics and professionals alike.
- But new research has showed that there is a possibility that the size effect was masked by other factors
- "Size Matters, If You Control Your Junk," Clifford S. Asness, Andrea Frazzini, Ronen Israel, Tobias J. Moskowitz, and Lasse Heje Pedersen, *Journal of Financial Economics* 129 (2018): 479-509
  - Investigate the size effect by adding a factor for firm quality to a multi-factor analysis of what drives stock prices --define quality as characteristics that investors should be willing to pay a higher price for, everything else equal --- defined their quality measure based on three categories: profitability, growth and safety.
  - Find that a key variable in explaining the changing size effect over time is the markets pricing of firm quality versus junk. They find that this relationship has a far stronger explanatory power than other factors (relationship of size to the market, value, or momentum).
  - Finding holds whether size is measured by market capitalization or non-market based ("fundamental") measures.
  - Finding holds for each of the 30 industries.

Size Factor Strengthens After Controlling for Quality

- "The Size Effect Continues to Be Relevant When Estimating the Cost of Capital," Roger J. Grabowski, *Business Valuation Review* 37(3) (2018).
  - Demonstrates that the size effect still exists after controlling for quality.

- These two papers differ in the way they define quality, but they reach the same conclusion:

The Size Effect exists after controlling for Quality.

Size Premia are correlated with underlying risks of companies

Companies Ranked by Market Value of Equity
Data for Year Ending December 31, 2019

Comparative Risk Characteristics **Exhibit C-1**
Data Smoothing with Regression Analysis
Dependent Variable: Average Unlevered Risk Premium
Independent Variable: Log of Average Market Value of Equity

Constant	17.573%
X Coefficient(s)	-2.655%

*Smoothed Unlevered Premium = 17.573% - 2.655% * Log(Market Value)*

Portfolio Rank by Size	Average Mkt. Value (in $millions)	Log of Average Mkt. Value	Number of Firms	Arithmetic Average Risk Premium	Average Debt to MVIC	Average Debt to Market Value of Equity	Average Unlevered Risk Premium[1]	*Smoothed Average Unlevered Risk Premium*	Beta (Sum Beta) Since '63	Average Unlevered Beta	Average Operating Margin	Average CV(Operating Margin)	Average CV(ROE)
1	300,185	5.48	40	6.06%	14.48%	16.93%	5.48%	*3.03%*	0.84	0.73	17.13%	9.63%	18.48%
2	71,668	4.86	37	4.89%	18.79%	23.14%	3.99%	*4.68%*	0.97	0.80	13.90%	11.14%	22.42%
3	42,250	4.63	33	6.11%	20.86%	26.36%	5.17%	*5.29%*	0.92	0.75	13.53%	11.63%	21.95%
4	28,567	4.46	35	6.85%	22.70%	29.37%	5.81%	*5.74%*	0.94	0.75	13.42%	12.55%	23.32%
5	21,223	4.33	29	6.10%	23.29%	30.35%	4.98%	*6.09%*	0.98	0.77	12.48%	12.94%	22.24%
6	16,018	4.20	37	7.53%	22.40%	28.86%	6.40%	*6.41%*	1.02	0.81	12.87%	13.64%	23.82%
7	12,575	4.10	38	7.20%	23.48%	30.68%	6.03%	*6.69%*	1.01	0.80	12.65%	13.41%	22.47%
8	10,256	4.01	33	8.40%	22.19%	28.52%	7.22%	*6.93%*	1.07	0.86	12.77%	14.37%	24.93%
9	8,292	3.92	32	9.42%	22.48%	28.99%	8.20%	*7.17%*	1.09	0.87	12.40%	14.30%	24.79%
10	7,226	3.86	30	8.98%	22.75%	29.45%	7.72%	*7.33%*	1.11	0.88	11.65%	14.74%	26.53%
11	6,044	3.78	33	8.42%	22.31%	28.72%	7.21%	*7.53%*	1.09	0.87	11.76%	14.76%	27.38%
12	4,960	3.70	41	8.36%	22.84%	29.60%	7.10%	*7.76%*	1.11	0.88	11.88%	14.80%	24.47%
13	4,288	3.63	36	8.71%	23.29%	30.36%	7.42%	*7.93%*	1.11	0.88	11.28%	15.55%	25.57%
14	3,678	3.57	40	8.90%	23.17%	30.16%	7.58%	*8.11%*	1.14	0.90	10.88%	17.07%	27.11%
15	3,148	3.50	40	10.61%	23.29%	30.36%	9.30%	*8.29%*	1.13	0.89	10.88%	17.18%	27.20%
16	2,802	3.45	34	10.21%	22.93%	29.76%	8.91%	*8.42%*	1.14	0.90	10.61%	17.77%	28.99%
17	2,487	3.40	43	10.39%	22.28%	28.66%	9.05%	*8.56%*	1.20	0.96	10.43%	18.41%	28.79%
18	2,140	3.33	48	8.38%	23.08%	30.01%	6.97%	*8.73%*	1.21	0.95	9.92%	19.68%	31.73%
19	1,724	3.24	44	9.77%	22.44%	28.92%	8.42%	*8.98%*	1.20	0.96	9.48%	21.36%	32.63%
20	1,431	3.16	52	10.31%	23.90%	31.40%	8.88%	*9.20%*	1.20	0.93	9.13%	22.71%	34.97%
21	1,209	3.08	47	9.70%	23.44%	30.62%	8.23%	*9.39%*	1.24	0.98	8.83%	23.93%	37.38%
22	951	2.98	62	10.59%	24.01%	31.60%	9.11%	*9.67%*	1.23	0.96	8.60%	24.86%	39.41%
23	719	2.86	57	10.11%	23.78%	31.20%	8.62%	*9.99%*	1.24	0.97	8.07%	26.55%	42.72%
24	463	2.67	92	12.96%	24.45%	32.36%	11.45%	*10.50%*	1.23	0.95	7.86%	30.32%	45.66%
25	148	2.17	209	16.72%	26.76%	36.54%	15.00%	*11.81%*	1.27	0.96	6.24%	43.78%	64.43%

CV(X) = Standard deviation of X divided by mean of X, calculated over 5 fiscal years.
[1] Unlevered risk premiums and unlevered betas are calculated using methodology described in Chapter 10 with an average assumed debt beta = 0.1.
Sources of underlying data: 1.) © 201702 CRSP®, Center for Research in Security Prices. University of Chicago Booth School of Business used with permission. All rights reserved. 2.) Morningstar *Direct* database. Used with permission. All rights reserved. Calculations performed by Duff & Phelps LLC.

Advances in Data Quality and Availability (cont'd)

- Summary
- Data sources are more sophisticated and available
- This evolution came about because of the computerization of underlying data.
- Interpretations of the data are much more dispersed because of the availability of data and the ability to analyze the data.
- Interpretation of data continues to evolve as anomalies in the market continue to develop
- But the many sources of alternative data has also led to easy misunderstanding and misinterpretations >> more data has not made the task of the valuation analyst easier. In fact, it is more complex today than ever.

Technology available to the analyst

Advances in Technology Available to the Analyst

- Finance books typically included tables of the Present Value of $1 and Present Value of an Annuity of $1 (see examples on the next two slides) because calculating PV was cumbersome.
- Wang Laboratories revolutionized electronic calculators by providing higher level math functions only previously available on very expensive computer systems. The 300-series (introductory price of $1,700) was introduced in 1968.
  - An option of a punch card reader was offered.
  - Offered a library of programs for applications in financial, statistics, and engineering function.
  - The Model 370 programmable keyboard added higher-level programmability to the base 300 series.
- Hewlett-Packard introduced the HP-35 scientific calculator and the HP-80 business calculator in 1973. Their initial prices were $395.
  - H-P introduced the HP12-C in 1981
- Texas Instruments SR-50 scientific calculator was introduced in 1973 for an initial price of $170. Business Analyst series of financial calculators was introduced in 1976.
- Note: Slide Rule production stopped in 1975.

Present Value of $1 from James Van Horne 3rd ed. (1968)

Table A-I

PRESENT VALUE OF ONE DOLLAR DUE AT THE END OF N YEARS

N	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	N
01	0.99010	0.98039	0.97007	0.96154	0.95238	0.94340	0.93458	0.92593	0.91743	0.90909	01
02	.98030	.96117	.94260	.92456	.90703	.89000	.87344	.85734	.84168	.82645	02
03	.97059	.94232	.91514	.88900	.86384	.83962	.81630	.79383	.77218	.75131	03
04	.96098	.92385	.88849	.85480	.82270	.79209	.76290	.73503	.70843	.68301	04
05	.95147	.90573	.86261	.82193	.78353	.74726	.71299	.68058	.64993	.62092	05
06	.94204	.88797	.83748	.79031	.74622	.70496	.66634	.63017	.59527	.56447	06
07	.93272	.87056	.81309	.75992	.71068	.65506	.62275	.58349	.54703	.51316	07
08	.92348	.85349	.78941	.73069	.67684	.62741	.58201	.54027	.50187	.46651	08
09	.91434	.83675	.76642	.70259	.64461	.59190	.54393	.50025	.46043	.42410	09
10	.90529	.82035	.74409	.67556	.61391	.55839	.50835	.46319	.42241	.38554	10
11	.89632	.80426	.72242	.64958	.58468	.52679	.47509	.42888	.38753	.35049	11
12	.88745	.78849	.70138	.62460	.55684	.49697	.44401	.39711	.35553	.31863	12
13	.87866	.77303	.68095	.60057	.53032	.46884	.41496	.36770	.32618	.28966	13
14	.86996	.75787	.66112	.57747	.50507	.44230	.38782	.34046	.29925	.26333	14
15	.86135	.74301	.64186	.55526	.48102	.41726	.36245	.31524	.27454	.23939	15
16	.85282	.72845	.62317	.53391	.45811	.39365	.33873	.29189	.25187	.21763	16
17	.84438	.71416	.60502	.51337	.43630	.37136	.31657	.27027	.23107	.19784	17
18	.83602	.70016	.58739	.49363	.41552	.35034	.29586	.25025	.21199	.17986	18
19	.82774	.68643	.57029	.47464	.39573	.33051	.27651	.23171	.19449	.16351	19
20	.81954	.67297	.55367	.45639	.37689	.31180	.25842	.21455	.17843	.14864	20
21	.81143	.65978	.53755	.43883	.35894	.29415	.24151	.19866	.16370	.13513	21
22	.80340	.64684	.52189	.42195	.34185	.27750	.22571	.18394	.15018	.12285	22
23	.79544	.63416	.50669	.40573	.32557	.26180	.21095	.17031	.13778	.11168	23
24	.78757	.62172	.49193	.39012	.31007	.24698	.19715	.15770	.12640	.10153	24
25	.77977	.60953	.47760	.37512	.29530	.23300	.18425	.14602	.11597	.09230	25

Source: Ezra Solomon, ed., *The Management of Corporate Capital,* (New York: The Free Press of Glencoe, Inc., 1959), pp. 313-16.

Present Value of an Annuity of $1 per year from James Van Horne 3rd ed. (1968)

Table A-2

PRESENT VALUE OF ONE DOLLAR PER YEAR. N YEARS AT R%

Year	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	Year
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	1
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355	2
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4868	3
4	3.9020	3.8077	3.7171	3.6299	3.5459	3.4651	3.3872	3.3121	3.2397	3.1699	4
5	4.8535	4.7134	4.5797	4.4518	4.3295	4.2123	4.1002	3.9927	3.8896	3.7908	5
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	6
7	6.7282	6.4720	6.2302	6.0020	5.7863	5.5824	5.3893	5.2064	5.0329	4.8684	7
8	7.6517	7.3254	7.0196	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	8
9	8.5661	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9852	5.7590	9
10	9.4714	8.9825	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4176	6.1446	10
11	10.3677	9.7868	9.2526	8.7604	8.3064	7.8868	7.4987	7.1389	6.8052	6.4951	11
12	17.2552	10.5753	9.9539	9.3850	8.8632	8.3838	7.9427	7.5361	7.1607	6.8137	12
13	12.1338	11.3483	10.6349	9.9856	9.3935	8.8527	8.3576	7.9038	7.4869	7.1034	13
14	13.0038	12.1062	11.2960	10.5631	9.8986	9.2950	8.7454	8.2442	7.7861	7.3667	14
15	13.8651	12.8492	11.9379	11.1183	10.3796	9.7122	9.1079	8.5595	8.0607	7.6061	15
16	14.7180	13.5777	12.5610	11.6522	10.8377	10.1059	9.4466	8.8514	8.3125	7.8237	16
17	15.5624	14.2918	13.1660	12.1656	11.2740	10.4772	9.7632	9.1216	8.5436	8.0215	17
18	16.3984	14.9920	13.7534	12.6592	11.6895	10.8276	10.0591	9.3719	8.7556	8.0214	18
19	17.2261	15.6784	14.3237	13.1339	12.0853	11.1581	10.3356	9.6036	8.9501	8.3649	19
20	18.0457	16.3514	14.8774	13.5903	12.4622	11.4699	10.5940	9.8181	9.1285	8.5136	20
21	18.8571	17.0111	15.4149	14.0291	12.8211	11.7640	10.8355	10.0168	9.2922	8.6487	21
22	19.6605	17.6580	15.9368	14.4511	13.1630	12.0416	11.0612	10.2007	9.4424	8.7715	22
23	20.4559	18.2921	16.4435	14.8568	13.4885	12.3033	11.2722	10.3710	9.5802	8.8832	23
24	21.2435	18.9139	16.9355	15.2469	13.7986	12.5503	11.4693	10.5287	9.7066	8.9847	24
25	22.0233	19.5234	17.4131	15.6220	14.0939	12.7833	11.6536	10.6748	9.8226	9.0770	25

Source: Solomon, *op. cit.,* pp. 317-20

Advances in Technology Available to the Analyst (cont'd)

Tables were published that showed the combinations of discount rates and growth rates that resulted in multiples:

Nicholas Molodovsky, Catherine May and Sherman Chottiner, "Common Stock Valuation: Principles, Tables and Application" (*Financial Analysts Journal*, Vol 21 (2), 1965)

"The tables in the appendix are stock valuation tables of a new type.

There exists an obvious need for stock value tables. Common stocks have no maturity. An infinite perspective is needed for their valuation. The interactions of future earnings and dividend growth rates weighted by time and rates of investment returns cannot be grasped by the unaided brain. Computer generated valuation tables serve as a focus which brings the complex stock investment factors into a single value figure.

Our Tables are based on a dividend model. Assuming long-term investment, dividends are the only returns from stocks; but future dividends cannot be estimated without estimating future earnings. To make the Tables operationally more effective, statistical relations were developed which allowed earnings projections to serve directly as the input.

Advances in Technology Available to the Analyst (cont'd)

"Attempts have been made by other analysts to construct stock value tables by imitating bond yield tables. Stocks were conceptually broken up into a series of payments over a period of years and a resale price. However. this approach calls for assumptions concerning rates of future dividend payouts and price at the date of the hypothetical future resale, thereby injecting additional unknowns into the problem. It also distorts the true nature of stocks.

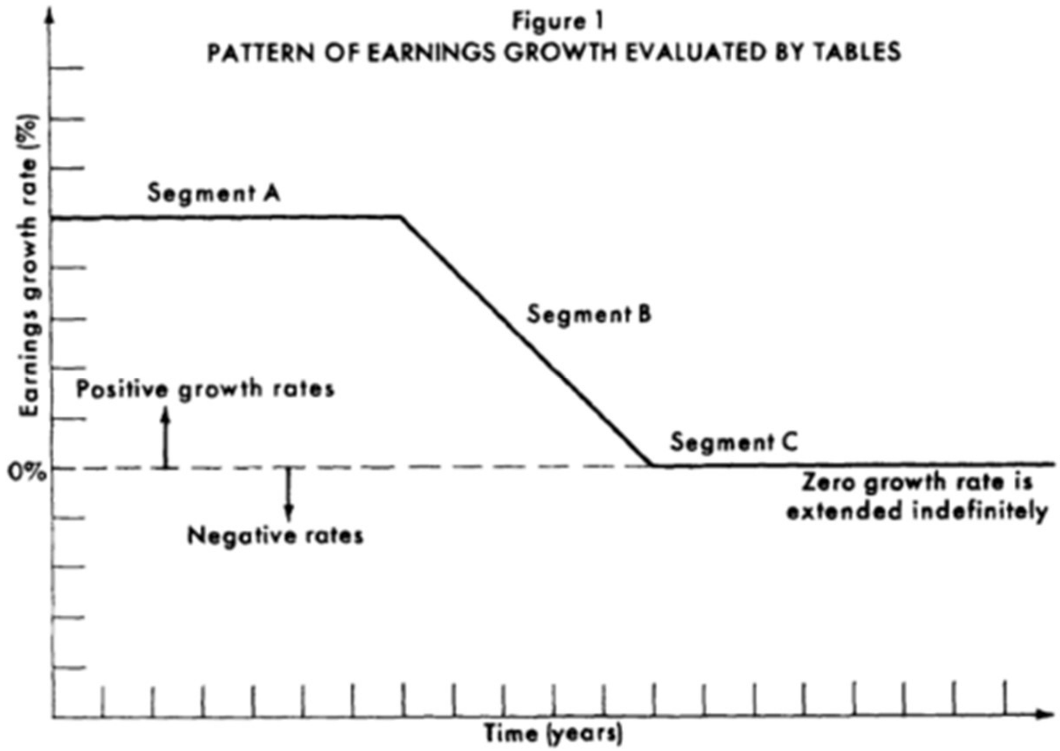
The theoretical structure of our Tables rests on the foundation that the value of a common stock is the present worth of its future stream of dividends. Mathematically, value (V) is:

$$V = D_0 + \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \dots + \frac{D_n}{(1+k)^n} + \dots$$

Where: D_0 is the dividend initially.
D_n is the dividend in the nth year.
k is the discount rate, or the desired rate of return.

The model assumes dividend projections taken out to infinity. Fortunately, economic infinity is not as forbidding as it may sound. This is because the discount factor becomes so large in the distant future that contributions to value become negligible.…The discount factor, *k*, is the desired rate of return on common stock investments (tables included *k* = 6% to 9%). Its setting by the investor should be guided by returns on alternative investments and historical stock market returns."

Molodovsky, May and Chottiner Tables



Molodovsky, May and Chottiner Tables (cont'd)

APPENDIX

Investment Values of Normal Earnings of $1 at 6% Return

Projected Earnings Growth Rate

1.0%

Years Constant Growth	YEARS DIMINISHING GROWTH 2	4	6	8	10	12	14	16	18	20
2	11.9	12.0	12.1	12.2	12.3	12.3	12.4	12.5	12.5	12.6
4	12.1	12.2	12.3	12.4	12.5	12.5	12.6	12.6	12.7	12.7
6	12.3	12.4	12.5	12.6	12.6	12.7	12.7	12.8	12.8	12.9
8	12.5	12.6	12.6	12.7	12.8	12.8	12.9	12.9	13.0	13.0
10	12.7	12.7	12.8	12.8	12.9	13.0	13.0	13.0	13.1	13.1
12	12.8	12.9	12.9	13.0	13.0	13.1	13.1	13.2	13.2	13.2
14	12.9	13.0	13.0	13.1	13.1	13.2	13.2	13.3	13.3	13.3
16	13.1	13.1	13.2	13.2	13.2	13.3	13.3	13.3	13.4	13.4
18	13.2	13.2	13.3	13.3	13.3	13.4	13.4	13.4	13.5	13.5
20	13.3	13.3	13.3	13.4	13.4	13.4	13.5	13.5	13.5	13.6
22	13.4	13.4	13.4	13.5	13.5	13.5	13.5	13.6	13.6	13.6
24	13.4	13.5	13.5	13.5	13.6	13.6	13.6	13.6	13.7	13.7
26	13.5	13.5	13.6	13.6	13.6	13.6	13.7	13.7	13.7	13.7
28	13.6	13.6	13.6	13.7	13.7	13.7	13.7	13.7	13.8	13.8
30	13.6	13.7	13.7	13.7	13.7	13.8	13.8	13.8	13.8	13.8

2.0%

Years Constant Growth	YEARS DIMINISHING GROWTH 2	4	6	8	10	12	14	16	18	20
2	12.2	12.4	12.6	12.7	12.9	13.1	13.2	13.3	13.5	13.6
4	12.6	12.8	13.0	13.1	13.3	13.4	13.6	13.7	13.8	13.9
6	13.0	13.2	13.4	13.5	13.6	13.8	13.9	14.0	14.1	14.2
8	13.4	13.6	13.7	13.8	14.0	14.1	14.2	14.3	14.4	14.5
10	13.8	13.9	14.0	14.2	14.3	14.4	14.5	14.6	14.7	14.8
12	14.1	14.2	14.3	14.5	14.6	14.7	14.8	14.9	15.0	15.0
14	14.4	14.5	14.6	14.7	14.8	14.9	15.0	15.1	15.2	15.3
16	14.7	14.8	14.9	15.0	15.1	15.2	15.3	15.3	15.4	15.5
18	14.9	15.0	15.1	15.2	15.3	15.4	15.5	15.5	15.6	15.7
20	15.2	15.3	15.3	15.4	15.5	15.6	15.7	15.7	15.8	15.9
22	15.4	15.5	15.5	15.6	15.7	15.8	15.8	15.9	16.0	16.0
24	15.6	15.7	15.7	15.8	15.9	16.0	16.0	16.1	16.1	16.2
26	15.8	15.8	15.9	16.0	16.0	16.1	16.2	16.2	16.3	16.3
28	15.9	16.0	16.1	16.1	16.2	16.3	16.3	16.4	16.4	16.5
30	16.1	16.2	16.2	16.3	16.3	16.4	16.4	16.5	16.5	16.6

3.0%

Years Constant Growth	YEARS DIMINISHING GROWTH 2	4	6	8	10	12	14	16	18	20
2	12.5	12.8	13.0	13.3	13.6	13.8	14.1	14.3	14.5	14.7
4	13.1	13.4	13.7	13.9	14.2	14.4	14.6	14.9	15.1	15.3
6	13.8	14.0	14.3	14.5	14.8	15.0	15.2	15.4	15.6	15.8
8	14.4	14.6	14.9	15.1	15.3	15.5	15.7	15.9	16.1	16.3
10	14.9	15.2	15.4	15.6	15.8	16.0	16.2	16.4	16.6	16.7
12	15.5	15.7	15.9	16.1	16.3	16.5	16.7	16.9	17.0	17.2
14	16.0	16.2	16.4	16.6	16.8	17.0	17.1	17.3	17.4	17.6
16	16.5	16.7	16.9	17.1	17.2	17.4	17.5	17.7	17.8	18.0
18	16.9	17.1	17.3	17.5	17.6	17.8	17.9	18.1	18.2	18.4
20	17.4	17.5	17.7	17.9	18.0	18.2	18.3	18.5	18.6	18.7
22	17.8	17.9	18.1	18.2	18.4	18.5	18.7	18.8	18.9	19.0
24	18.1	18.3	18.5	18.6	18.7	18.9	19.0	19.1	19.2	19.3
26	18.5	18.7	18.8	18.9	19.1	19.2	19.3	19.4	19.5	19.6
28	18.8	19.0	19.1	19.3	19.4	19.5	19.6	19.7	19.8	19.9
30	19.2	19.3	19.4	19.6	19.7	19.8	19.9	20.0	20.1	20.2

Projected Earnings Growth Rate

4.0%

Years Constant Growth	YEARS DIMINISHING GROWTH 2	4	6	8	10	12	14	16	18	20
2	12.7	13.1	13.5	13.9	14.3	14.6	15.0	15.3	15.6	15.9
4	13.7	14.0	14.4	14.8	15.1	15.5	15.8	16.1	16.4	16.7
6	14.6	14.9	15.3	15.6	16.0	16.3	16.6	16.9	17.2	17.5
8	15.4	15.8	16.1	16.5	16.8	17.1	17.4	17.7	18.0	18.3
10	16.2	16.6	16.9	17.3	17.6	17.9	18.2	18.4	18.7	19.0
12	17.0	17.4	17.7	18.0	18.3	18.6	18.9	19.2	19.4	19.7
14	17.8	18.1	18.4	18.7	19.0	19.3	19.6	19.9	20.1	20.3
16	18.5	18.9	19.2	19.5	19.7	20.0	20.3	20.5	20.8	21.0
18	19.3	19.6	19.9	20.1	20.4	20.7	20.9	21.2	21.4	21.6
20	19.9	20.2	20.5	20.8	21.0	21.3	21.5	21.8	22.0	22.2
22	20.6	20.9	21.2	21.4	21.7	21.9	22.1	22.4	22.6	22.8
24	21.2	21.5	21.8	22.0	22.3	22.5	22.7	22.9	23.1	23.3
26	21.9	22.1	22.4	22.6	22.8	23.1	23.3	23.5	23.7	23.9
28	22.4	22.7	22.9	23.2	23.4	23.6	23.8	24.0	24.2	24.4
30	23.0	23.2	23.5	23.7	23.9	24.1	24.3	24.5	24.7	24.9

5.0%

Years Constant Growth	YEARS DIMINISHING GROWTH 2	4	6	8	10	12	14	16	18	20
2	13.0	13.5	14.0	14.5	15.0	15.5	15.9	16.4	16.8	17.2
4	14.2	14.7	15.2	15.7	16.2	16.6	17.1	17.5	17.9	18.4
6	15.4	15.9	16.4	16.8	17.3	17.8	18.2	18.6	19.0	19.4
8	16.5	17.0	17.5	18.0	18.4	18.9	19.3	19.7	20.1	20.5
10	17.6	18.1	18.6	19.1	19.5	19.9	20.4	20.8	21.2	21.6
12	18.8	19.2	19.7	20.1	20.6	21.0	21.4	21.8	22.2	22.6
14	19.8	20.3	20.8	21.2	21.6	22.1	22.5	22.9	23.2	23.6
16	20.9	21.4	21.8	22.2	22.7	23.1	23.5	23.9	24.2	24.6
18	22.0	22.4	22.8	23.3	23.7	24.1	24.5	24.9	25.2	25.6
20	23.0	23.4	23.9	24.3	24.7	25.1	25.5	25.8	26.2	26.6
22	24.0	24.4	24.8	25.3	25.7	26.0	26.4	26.8	27.1	27.5
24	25.0	25.4	25.8	26.2	26.6	27.0	27.4	27.7	28.1	28.4
26	25.9	26.4	26.8	27.2	27.5	27.9	28.3	28.6	29.0	29.3
28	26.9	27.3	27.7	28.1	28.5	28.8	29.2	29.5	29.9	30.2
30	27.8	28.2	28.6	29.0	29.4	29.7	30.1	30.4	30.8	31.1

6.0%

Years Constant Growth	YEARS DIMINISHING GROWTH 2	4	6	8	10	12	14	16	18	20
2	13.3	13.9	14.6	15.2	15.8	16.4	17.0	17.6	18.1	18.7
4	14.6	15.2	15.9	16.5	17.1	17.7	18.3	18.9	19.4	20.0
6	15.9	16.5	17.2	17.8	18.4	19.0	19.6	20.2	20.7	21.3
8	17.2	17.9	18.5	19.1	19.7	20.3	20.9	21.5	22.1	22.6
10	18.5	19.2	19.8	20.4	21.0	21.6	22.2	22.8	23.4	23.9
12	19.8	20.5	21.1	21.7	22.3	22.9	23.5	24.1	24.7	25.2
14	21.1	21.8	22.4	23.0	23.7	24.3	24.8	25.4	26.0	26.6
16	22.5	23.1	23.7	24.4	25.0	25.6	26.2	26.7	27.3	27.9
18	23.8	24.4	25.0	25.7	26.3	26.9	27.5	28.0	28.6	29.2
20	25.1	25.7	26.4	27.0	27.6	28.2	28.8	29.4	29.9	30.5
22	26.4	27.0	27.7	28.3	28.9	29.5	30.1	30.7	31.2	31.8
24	27.7	28.3	29.0	29.6	30.2	30.8	31.4	32.0	32.5	33.1
26	29.0	29.7	30.3	30.9	31.5	32.1	32.7	33.3	33.9	34.4
28	30.3	31.0	31.6	32.2	32.8	33.4	34.0	34.6	35.2	35.7
30	31.6	32.3	32.9	33.5	34.1	34.7	35.3	35.9	36.5	37.0

Soldofsky and Murphy Tables

Two sets of tables were published that showed the combinations of discount rates and growth rates that resulted in multiples:

R.M. Soldofsky and J.T. Murphy, *Growth Yields on Common Stock: Theory and Tables* (Iowa City: Bureau of Business and Economic Research, State University of Iowa, 1963)

- Published extensive tables of dividend "multipliers" for various discount rates (or "growth yields") and various constant estimated growth rates of dividend payment streams (ranging from 2% to 12%) over selected periods of time (5,10,15,20,25,35,50 and 75 years).
- Second set of tables of dividend "multipliers" for two-step dividend growth rates, where dividends are assumed to grow at a high rate for one period of years and a lower rate for a second period of years.
  - These combinations of rates range from 15%/3% to 15%/6% for various packages of time periods. Long-high-growth/short-lower-growth packages are included along with short-high-growth/long-lower-growth packages for total length periods of 10 to 50 years.
- The analyst can use the tables to find the discount rate that equates the expected steam of dividends with the market price of the stock.
  - This is the methodology of the two-stage implied cost of capital.

Soldofsky and Murphy Tables (set 1)

Multipliers for Each Dollar of Most Recent Period Payments on a Common Stock Given a Certain Growth Yield (Discount Rate), a Constant Rate of Growth for the Period Indicated, and a Zero Perpetual Rate of Growth Thereafter

Years	6½%	6¾%	7%	7½%	8%	9%	10%
	Dividend Growth Yields[a]						
	Dividend Growth Rate 3%						
5	17.51	16.86	16.25	15.15	14.20	12.60	11.31
10	19.34	18.58	17.84	16.64	15.56	13.73	12.27
15	20.88	20.04	19.20	17.84	16.61	14.58	12.96
20	22.17	21.24	20.38	18.81	17.45	15.21	13.44
25	23.28	22.25	21.30	19.58	18.12	15.70	13.80
35	25.00	23.79	22.69	20.73	19.04	16.33	14.24
50	26.66	25.26	23.98	21.73	19.82	16.81	14.53
75	28.03	26.42	24.95	22.42	20.33	17.06	14.67
	Dividend Growth Rate 4%						
5	18.29	17.61	16.96	15.81	14.81	13.14	11.79
10	20.88	20.06	19.30	17.93	16.74	14.74	13.15
15	23.19	22.23	21.34	19.73	18.33	16.01	14.17
20	25.24	24.12	23.09	21.25	19.64	17.01	14.95
25	27.05	25.79	24.61	22.53	20.73	17.80	15.53
35	30.07	28.50	27.08	24.53	22.38	18.92	16.30
50	33.41	31.42	29.63	26.52	23.93	19.87	16.89
75	36.65	34.16	31.94	28.17	25.11	20.49	17.22

[a] Growth yields for rates up to and including 4% are based upon discounting the expected income stream for 200 years. A 100-year income stream was used in determining growth yields beginning with 4¼% through 10%, and a 50-year stream was used for 12 per cent and above. In all cases, however, at least 99 per cent of the value of a perpetuity is included at each growth yield. In most cases much of the final 1 per cent of the value of a perpetuity is included despite the foreshortening of the discount period.

Source: R. M. Soldofsky and J. T. Murphy, *Growth Yields on Common Stock: Theory and Tables* (Iowa City: Bureau of Business and Economic Research, State University of Iowa, 1963), p. 66.

Soldofsky and Murphy Tables (set 2)

Multipliers for Each Dollar of Most Recent Period Payments on a Common Stock Given a Certain Growth Yield (Discount Rate) and Differing Rate of Growth for Each of Two Periods Indicated and a Zero Perpetual Rate of Growth Thereafter*

Growth Periods—Years			Dividend Growth Yields[a]					
First Period	Second Period	Total	6½%	6¾%	7%	8%	9%	10%
			Growth Rates 15%, 6%[b]					
5	5	10	35.55	34.08	32.71	28.10	24.53	21.70
5	10	15	42.08	40.20	38.45	32.60	28.12	24.59
10	5	15	58.49	55.70	53.13	44.52	37.96	32.82
5	15	20	48.45	46.10	43.93	36.70	31.24	26.99
10	10	20	68.06	64.57	61.35	50.68	42.64	36.43
15	5	20	92.11	87.04	82.38	66.99	55.50	46.71
5	20	25	54.66	51.78	49.14	40.43	33.94	28.99
10	15	25	77.39	73.12	69.19	56.28	46.71	39.44
15	10	25	106.14	99.88	94.15	75.41	61.62	51.22
5	30	35	66.60	62.53	58.82	46.90	38.35	32.03
10	25	35	95.32	89.26	83.74	66.01	53.33	44.00
15	20	35	133.09	124.15	116.04	90.03	71.56	58.08
20	15	35	181.90	168.92	157.16	119.70	93.37	74.36
5	45	50	83.22	77.09	71.60	54.55	43.00	34.91
10	40	50	120.31	111.15	102.94	77.50	60.33	48.33
15	35	50	170.64	157.05	144.89	107.30	82.08	64.59
25	25	50	328.42	299.39	273.52	194.49	142.58	107.43
			Growth Rates 15%, 10%[b]					
5	5	10	40.89	39.13	37.51	32.03	27.80	24.46
5	10	15	55.01	52.36	49.92	41.75	35.54	30.70
10	5	15	66.31	63.03	60.00	49.90	42.23	36.26
5	15	20	71.58	67.71	64.15	52.40	43.64	36.94
10	10	20	87.00	82.20	77.77	63.20	52.34	44.06
15	5	20	103.57	97.66	92.22	74.34	61.09	51.01
5	20	25	91.02	85.51	80.46	64.06	52.11	43.18
10	15	25	111.29	104.43	98.15	77.76	62.93	51.85
15	10	25	133.90	125.42	117.67	92.53	74.30	60.74
5	30	35	140.50	130.03	120.58	90.80	70.24	55.66
10	25	35	173.08	160.02	148.24	111.15	85.57	67.43
15	20	35	211.07	194.86	180.23	134.23	102.58	80.20
20	15	35	254.56	234.54	216.49	159.86	121.04	93.72
5	45	50	250.51	226.25	204.84	140.93	100.59	74.31
10	40	50	310.47	280.19	253.48	173.76	123.47	90.73
15	35	50	382.66	344.93	311.65	212.43	149.91	109.29
25	25	50	570.70	512.64	461.48	309.45	214.28	152.96

* See notes on facing page.

Advances in Technology Available to the Analyst (cont'd)

- In 1970 the only computers generally available were main-frame (e.g., IBM 360) and mini-computers (e.g., HP). Remember, Apple was not founded until 1976 and IBM did not sell its first microcomputer (MS-DOS operating system) until April 1986.
- Users needed to own a mainframe or minicomputer or rent time on a time-share service.
- Software was available that was customized. For example, Alfred Rapport introduced *Alcar*, a modeling and valuation software program that had limited flexibility in formatting, in the early 1970's.
- Spreadsheet software:
  - LANPAR (1969)(LANguage for Programming Arrays at Random) - first spreadsheet application, used on mainframes and timeshare services
  - VisiCalc (1979) - first microcomputer program- initially for HP-125, Sharp MZ80; then Apple II.
  - SuperCalc (1980) – first for Osborne 1; then MS-DOS in 1982.
  - Multiplan (1982) - first DOS spreadsheet program
  - Lotus 1-2-3 (1983) - introduced on DOS operating system microcomputers
  - Microsoft Excel (1987) – and the "rest is history"
- Bloomberg Terminal (1981): market data and tools

Acceptance of Evolving Valuation Theory

Acceptance of Evolving Valuation Theory

Acceptance of changes in financial theory takes time

- STB replaced the single-stage DCF model with CAPM in January 2008. Also adopted a multi-stage DCF in 2009.
- FERC adopted a two-stage DCF model in 2014.
- Delaware Chancery Court:
  - DCF accepted as an accepted valuation method. n *Weinberger vs. UOP* 457 A.2d 701 (1983)
  - CAPM was first introduced Cede & Co. v. Technicolor, 1990 Del.Ch. LEXIS 259 (Oct. 19, 1990)
- U.S. Tax Court accepted DCF and CAPM in *The Northern Trust Company,* 87 T.C. 349; 1986 U.S. Tax Ct. LEXIS 68; 87 T.C. No. 21, August 11, 1986, Filed. (Excerpt included herein)

- Acceptance has evolved as the practices have become more widely used. Practitioners first learn the theory, but adopting it typically meets resistance of superiors who are comfortable with earlier practices. When practitioners become the leaders, they are in a position to use the theories with which they learned and seem to match risk and return better.
- What will the next 50 years bring?

DISCOUNT RATE DEVELOPMENT in U.S. TAX COURT

- **Excerpt from the Decision in:**
- **The Northern Trust Company, Transferee and Trustee; Arthur L. Simon, Transferee and Trustee; and Jeffery J. Simon, Transferee and Trustee, et al., Petitioners v. Commissioner of Internal Revenue, Respondent, 87 T.C. 349; 1986 U.S. Tax Ct. LEXIS 68; 87 T.C. No. 21, August 11, 1986, Filed**
- Judge: Nims, Valuation date: May 7, 1976, Report dated: December 1, 1984

Grabowski, however, used the discounted cash-flow approach to value C/S/K and Holland **[the two operating businesses]**. This valuation method is based on the assumption that the price an investor will pay for a share of stock is the present value of the future stream of income he expects to receive from the investment.

DISCOUNT RATE DEVELOPMENT in U.S. TAX COURT (cont'd)

Under the discounted cash-flow approach, Grabowski determined a present value equivalent for: (1) The amount of cash that C/S/K and Holland could pay to their shareholders over a 10-year period without impairing business operations (we will hereinafter refer to this amount as available cash-flow), [n17] and (2) the companies' residual value at the end of 10 years. Adding these two values together, Grabowski determined a second value for the common shareholders' equity in C/S/K and Holland.

- - - - - - - - - - - - - - Footnotes - - - - - - - - - - - - - - -

n17 Grabowski defined available cash-flow as: (1) Net income after taxes *plus* (2) depreciation/amortization *minus* (3) increases in net working capital needed to support sales *minus* (4) capital expenditures *minus* (5) repayments of principal on long-term debts *minus* (6) preferred stock dividends. In his actual computations of available cash-flow, Grabowski also made an addition and a reduction in net working capital for certain years. See table summarizing Grabowski's computation *infra*.

DISCOUNT RATE DEVELOPMENT in U.S. TAX COURT (cont'd)

To determine a present value equivalent for these 10-year projected cash-flows, Grabowski was required to determine an appropriate discount rate. He used the following formula to arrive at the appropriate discount rate:

K = R[f] + Beta (R[1]) + R[2]

K = cost of equity capital

R[f] = current market rate on U.S. Government bonds

R[1] = premium an investor would expect before he would invest in common stock rather than U.S. Government bonds

Beta = relationship between the movement of stock prices for companies engaged in specific industries and the movement of stock prices in general

R[2] = additional premium an investor would expect before he would invest in the common stock [of C/S/K and Holland

DISCOUNT RATE DEVELOPMENT in U.S. TAX COURT (cont'd)

Grabowski determined that in May 1976, the current market yield on 10-year U.S. Government bonds was 6.96 percent.

To this figure he added a risk premium which represented the additional return an investor would require before he would invest in common stocks of publicly traded companies engaged in the construction business and railroad supply business rather than U.S. Government securities. To determine this premium, Grabowski first determined that an investor would require an additional return of 6 to 8 percent before he would invest in common stocks in general, rather than U.S. Government securities. Grabowski based this figure on studies which found that investors in diversified portfolios of common stocks trading on the New York Stock Exchange realized yields which were 6 percent to 8 percent larger than yields on long-term U.S. Government securities.

DISCOUNT RATE DEVELOPMENT in U.S. TAX COURT (cont'd)

Grabowski then multiplied this risk premium by a "beta coefficient" which represented the relationship between the movement of stock prices for publicly traded companies engaged in the construction business and the railroad supply business and the movement of stock prices in general. Based on a sample of common stock of publicly traded companies engaged in construction, Grabowski estimated that such stocks had a beta of 1.3 (i.e., when stock prices in general increase or decrease 10 percent, the stock prices for publicly traded companies engaged in construction increase or decrease 30 percent). Based on a sample of common stocks of publicly traded companies engaged in the railroad supply business, Grabowski determined that such stocks had a beta of .95 (i.e., when stock prices in general increase or decrease by 10 percent, the stock prices for publicly traded companies engaged in the railroad supply business increase or decrease 9.5 percent).

DISCOUNT RATE DEVELOPMENT in U.S. TAX COURT (cont'd)

Finally, Grabowski added a risk premium to reflect the additional return an investor would require before he would choose to invest in the common stock of an unlisted company similar to C/S/K and Holland. He determined that a risk premium of 3 percent to 6 percent would be appropriate for Holland and a risk premium of 4 percent to 7 percent would be appropriate for C/S/K.

Applying the formula K = R[f] + Beta (R[1]) + R[2] to the above variables, Grabowski determined that an investor would require an expected average return of 21 percent on an investment in the common stock of C/S/K and Holland before he would decide to make such an investment. Grabowski applied the formula as follows:

C/S/K: K = 6.96% + 1.3 (6%-8%) + (4%-7%)

Holland: K = 6.96% + .95 (6%-8%) + (3%-6%)

THANK YOU

QUESTIONS?

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ABOUT THE COST OF CAPITAL NAVIGATOR

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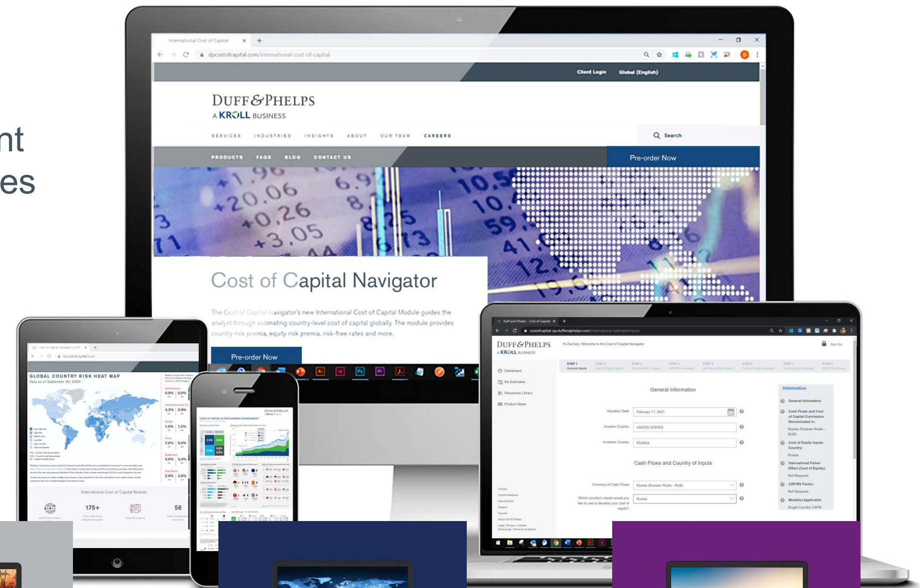

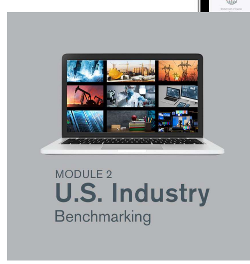

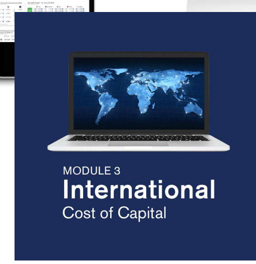

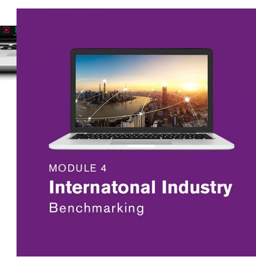

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